Ugo Broggi, a Neglected Precursor
in Modern Mathematical Economics

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ABBREVIATIONS IN THE MAIN TEXT:
AES: Academy of Economic Science
FES: Faculty of Economic Sciences (University of Buenos Aires)
IHCS: Institute of Higher Commercial
NULP: National University of La Plata
UBA: University of Buenos Aires

Argentine culture was deeply pervaded by European works. The fact reveals, on the one hand, the open-mindedness of Argentine scholars to receive messages on science and arts. And it also shows that reception of foreign influences was usually followed by some kind of active processing, either in developing, enriching, criticising or rejecting the original messages. The story to be told in this paper refers to a man who never took up with some new idea without adding to it some own reflection and sometimes a significant contribution: Hugo Broggi. We start from a biographical reconstruction, then to his academic activities in Argentina, and finally we proceed to review some of his contributions to modern economic analysis.

I - Life sketch

Ugo Broggi was born in Italy on 20 December 1880, the son of Ettore Broggi and Ester Meotti. He studied in Italy and Germany,[2] graduating first in Actuarial Science (1902), then in Economic Science (1904).

In the latter year Broggi began to collaborate with the ***Giornale degli Economisti***, the most important Italian economic journal, with which he kept connected for at least two decades. He also published in Bolletino dell'Associazione degli Attuari italiani, Zeitschrift für die gesammte Versicherungswissenschaft, Enseignement Mathématique, Rendiconti del Circolo Matematico di Palermo, The American Mathematical Monthly, Zeitschrift für angewandte Mathematik und Mechanik, Revista de la Universidad de Buenos Aires, Anuario de la Facultad de Ciencias Económicas, etc.[3]

Early in his career published ***Matematica Attuariale*** (Teoria statistica della mortalità. Matematica delle assicurazioni sulla vita) (1906), translated into French (***Theorie des assurances sur la vie***,1907) and into German (1911).

In 1907 Broggi received his doctorate at Göttingen, with a dissertation on ***Die Axiome der Wahrscheinlichkeitsrechnung***. In the same year he got a Ph.D. in Philosophy.

In 1910 arrived at Buenos Aires, where he would settle for seventeen years.[4] On 22 March 1910 a note was sent to the Ministry of Justice and Public Education with a list of three candidates for the chair of Financial Mathematics at the IHCS: 1st Ugo Broggi, 2nd Claro C. Dassen, 3rd Manuel Ordóñez. Broggi was the professor elected.

Appointed professor of *Mathematical Analysis* (1911) and *Higher Mathematics* (1912) at the NULP, he was *the first who taught that subject* [Higher Mathematics] *in Argentina from modern viewpoints*.[5] Accordingly, De Cesare ranked Broggi on a par with Rey Pastor as

marking a turning point in Argentine mathematical studies. On 17 June 1912 the Council of the UBA, after nomination by the IHCS (June 3), appointed him full professor of *Statistics*.

On the basis of the IHCS, the FES was created on 9 October 1913. On 18 November 1913 the Council of the UBA elected him to the first Council of the FES for a term of six years. On 7 November 1914 Broggi was elected member of the Academy of Economic Science[6], created on 14 November 1914. On 28 November the first curriculum was passed, which included 28 subjects, among them three on mathematics and thirteen on economic studies. Of course Broggi was confirmed as professor of Statistics, being his assistant professor Eng. Alejandro E. Bunge.

In the session of 27 November 1917 the Chamber of Deputies elected Prof. Broggi and Prof. Bunge to study the creation of a Pension Fund for railway employees. That activity mixed applied economics and actuarial mathematics.

Prof. Cañellas remembered that Broggi, in his lectures on Statistics, worked enthusiastically at the blackboard, followed by his pupils, who were but six or seven.[7]

On 20 May 1920 Broggi was appointed to lecture the course on Financial Mathematics (1st course). He is also elected member of the editorial staff of **Revista de Ciencias Económicas**, for the periodo 1 July 1921 to 30 April 1922. On 8 March 1922 the Council of the FES confirmed Broggi as professor of Financial Mathematics, to succeed prof. Casariego, recently died. On 18 June 1924 Broggi resigned his post as councillor, on behalf of some journalistic articles. The resignation was not accepted.

In 1924 it was founded the Argentine Mathematical Society, and Broggi was among the founding members, in a list which included the names of the first pioneers of Argentine mathematical research: J. Babini, J. Blaquier, E. A. de Cesare, F. La Menza, A. Valeiras, J.C. Vignaux and of course J. Rey Pastor.[8]

At the end of 1924 Broggi decided to return to his motherland, whence he was absent for almost fifteen years, and visit other European nations. In view of such decision, the Dean Mario Sáenz entrusted him, in his twofold character of councillor and professor, to represent the FES before the universities and institutes that he might visit, to arrange the exchange of their productions with those of the Seminar on Economics and Finance, and lastly to study the organisation of statistical and actuarial institutes. On 3 December 1924 Broggi accepted that commitment. On March 5 he addressed a letter to the Dean asking for permission to take leave for the year 1925. The letter, although dated from Rome, indicated as address 'Milano, Via Boscovich 27'. The same address occurs in a letter of 29 March 1926, in which Broggi informed an illness, followed by his mother's death. He could not resume his lectures at Buenos Aires up to 1 June 1926. In that year Broggi taught two subjects at the UBA, namely Financial Mathematics and Statistics, the latter with the assistance of Eng. Alejandro E. Bunge.

In 1927 Broggi gave in Rosario some lectures on mathematics. Those lectures were attended by the so-called 'Argentine Pearson', the would-be eminent statistician Carlos E. Dieulefait. The latter started that year his teaching activity at the University of the Litoral (Rosario). His intellectual debt with Broggi was recognised by him in the following terms: 'the courses designed under Broggi's influence 'contained a mix of statistical methods and applications to several fields, especially economics' (Dieulefait 1977: 2).

Again at the end of 1927 Broggi intended to leave Argentina to Europe. The Council of the FES availed itself of that opportunity to entrust Broggi the representation of the FES, and in that character to get in touch with John Maynard Keynes and Giorgio Mortara, to invite them to lecture four-month courses (from June to September) at the Faculty during 1928 or 1929.

He was also commended to study and report the FES about the organisation of actuarial studies in Germany and Great Britain.

On 6 March 1928 Broggi wrote from Milano ('Via Boscovich 27') to the Dean of FES: "I have recently enjoyed a too long leave, not to dare to ask for another, and for a whole year, which I intend to devote to study. Therefore I resign to the chairs that I performed at the Faculty, in conditions that allowed me to forget my condition of foreigner. And doing so I send to all my colleagues, from the Dean to the youngest of my disciples, the expression of an affectionate friendship, that surely will last while I live".

Broggi's chairs were entrusted to Argentino V. Acerboni (Satistics) and Justo Pascali (Mathematics, 1st course), respectively. On 5 June 1928 Broggi was granted a leave for the year 1928. Broggi thanked in a letter dated 'Parma, 7 May 1928', with the addition of 'Milano, Via Boscovich 27' in HIS own hand, written on stationery from 'P.N.F. Gruppo Nazionale Fascista della Scuola SEZIONE UNIVERSITARIA DI PARMA' .

By September 1928 the FES sent Broggi a radio-telegram offering him to concentrate on Actuarial Mathematics and leave Statistics. On 13 September he accepted the proposal, and on 6 October he was officially appointed by the UBA. But Broggi did not return to Buenos Aires, and on 25 February 1929 (again from 'Milano, Via Boscovich 27' [9]) wrote to the Dean: "I have been delaying my letter beyond what is proper, because writing and sending it meant to me to take a definitive decision, which anyhow would be burdensome. But the wish of devoting myself to study, free from teaching duties, leads me finally to insist in resigning to the chairs of mathematics and actuarial mathematics, that I presented last year, and to take farewell of my colleagues and disciples at the Faculty". On 18 April the Council turned down Broggi's resignation, on the ground thaat "his services are indispensable for the success of the teaching given by this house", and granted him a leave for the year 1929. Still on 11 July 1929 the Council of the FES gave Broggi the degree of Actuary, taking into account his previous studies and his teaching. Broggi delayed more than one month to answer, for he was outside Milan. His letter (from 'Milano, Via Francesco Sforza 41') thanked for the diploma of Actuary. But on 4 March 1930 [10] insisted in resigning to his courses, since the conditions that motivated his previous resign had not change. In view of Broggi's insistence, the Council accepted his resignation to both chairs (Mathematics, Actuarial Mathematics) on the meeting of 24 April 1930, on the following grounds: "With deep sorrow, the Faculty's authorities and in particular myself (i.e. the Dean Santiago B. Zaccheo), see to go far away from it one of its more illustrious masters; owner of a vast background, excellent teaching conditions and outstanding personal qualities. You have contributed with true success to consolidate the prestige of the Faculty during the most difficult moments of its existence, no only by the fitness of your lectures, but also the efficacy of your administrative activity along the various instances in which you integrated the Council. Your retirement from the chair does not mean, in any sense, a break with this house, in which you will always be remembered with affection and admiration, by your colleagues and former pupils. Besides, in accordance with the Ordinance approved by the Council, your name will always appear in the roll of teaching personnel of the Faculty".

For the period 1930-45 I have got no bibliographical data on Broggi.

After the end of World War II he returned for a short while to Buenos Aires, and in that occasion his former pupils rendered him a tribute in a pleasant party, where Broggi was courageous enough to acknowledge that he too had been wrong when -referring to his motherland- he had believed in the possibility of the agency of some providential man. [11]

Ugo Broggi died 27 days before his 85th anniversary, on 23 November 1965.

II - Teaching mathematics [12]

At the University of La Plata, Broggi taught Mathematical analysis (first course): Fundamental concepts of analytic geometry and differential and integral calculus. Applications of calculus to geometry and rational mechanics. Second course: Special theories of differential and integral calculus. Numerical methods of differentiation and integration. Integration of ordinary differential equations. Applications to geometry and rational mechanics. Broggi taught both since 1921, and as a result of both courses he wrote his lectures, afterwards published by the Faculty under the title ***Mathematical Analysis*** (Vol.I: General Theories. Functions of more than one variable).

Broggi conducted several courses on Theory of Functions and Integral Equations, during the years 1925, 1926 and 1927. After 1923 taught Real Functions and Analytic Functions.

Broggi taught several courses on Higher Anaysis: during 1921, Differential Equations and Eliptical Functions. After 1925, Analytic Functions. He also conducted courses on Integral Equations. In 1914 published ***Linear Integral Equations***, the first book on the subject published in Argentina.

Although Broggi was mainly analyst, not geometer, in his time he was the only professor of higher mathematics, and had to give the lectures on Geometry. He taught Differential Geometry (1923) with the following contents: Differential geometry of 2-dimension manifolds. Geodesics of a surface. Parallelism and curvature on a V3. Tensors. Covariant differentiation. Riemann's symbols. Riemann's tensor. Christoffel. Gravity field. General principles of dynamics. Elements of calculus of variations. The principles of Hamilton and minimal action. Lagrange equations. Canonical equations of Hamilton.

He also taught Algebraic Geometry (1924) with the following contents: Jacobians. Discriminants and resultants. Homogeneous Cartesian Co-ordinates. Projective co-ordinates of the 1st, 2nd and 3rd species. General theory of polars. Classes of a curve. Points in infinity of algebraic curves. Plucker's formulae. Gender of an algebraic curve. Integrals of algebraic functions. Curves of gender one and two. Affine and orthogonal projective groups.

III - Teaching statistics

Broggi has been considered the founder of mathematical statistics in Argentina by no less an authority than Carlos E. Dieulefait. He stated Broggi's influence on statistical studies as follows:

> in Argentina some significant activity in university teaching began to develop when the Faculty of Economic Science of the University of Buenos Aires appointed to the chair of Statistics the prestigious mathematician and actuary doctor Hugo Broggi.

> (Dieulefait 1977: 2)

We may know the contents of Broggi's teaching by the programs of his courses, of which those corresponding to 1913-1924 (one for each year) still exist in his personal file, kept in the Archive of the FES. The program for 1913 had the following contents: I. *Mathematical Introduction*. 1. Graphical representations. 2. Elements of difference-differential calculus. 3. Least squares. 4. Interpolation theory. 5. Elements of probability theory. II. *General Part*. *a*) Theory of attributes: 1. Association theory. 2. Partial association. 3. Contingency tables. *b*) Theory of variables: 1. Frequency curves. 2. Averages or index numbers. 3. Dispersion measures. 4. Correlation theory. 5. Partial correlation. *c*) Inductive method and statement of empirical laws. *d*) Application of probability theory to statistics: 1. Normal dispersion. 2. Dispersion coefficients. 3. Extension of the law of errors. III. *Special Part*. *a*) Methods of

biometric statistics: 1. Formal theory of population. 2. Biometric functions. 3. Foundations of insurance theory and pension funds. *b*) Economic statistics.

The reading assignment for 1919 included the books of K. Pearson, ***Grammar of Science***; Lexis, ***Abhandlungen***; Westergaard, ***Die Grundzüge der Statistik***; Mortara, ***Elementi di Statistica***; King, ***Elements of statistical method***; Bowley, ***Elements of statistics***; Yule, ***An introduction to the theory of statistics***; I. Fisher, ***The purchasing Power of Money*** (1911); Elderton, ***Frequency curve and correlation***; Lenoir, ***Etudes sur la formation et le mouvement des prix***; Lexis, ***Einbertung in die Bevolkerungs statistik***; Broggi, ***Theorie des assurances sur la vie***; Benini, ***Principii di demografia***; Eotento, ***Statistica***.

Broggi also did research on Statistics, and some of his publications are specific to this field, such as 'On a formula for quadrature' (1922), 'Ueber mechanische Ausgleichung' (1925), 'On interpolation' (1926), 'On a new formula for fitting', etc. Broggi (1925) developed a new formula for fitting a set of observations, which had been tried at the FES to produce new estimates of mortality tables.

IV - Teaching financial mathematics

In 1920 Broggi succeeded Eng. Orfilio Casariego as professor of Financial Mathematics (1st course) at the FES. The Program dated November 1923 gives us an idea of his teaching: **1** - Cartesian co-ordinates - Equation of the straight line - Particular problems - Angle of two lines - Tangent and normal to a curve.- **2** - Graphical representation of a function - Second degree curves: parabola, ellipse and hyperbola - General equation of the circle.- **3** - Trigonometric functions, sine, cosine, tangent - Theorems of sine and cosine - Theorems of addition - Angular measure in radians - Limit h = 0 for (sin h)/h = 1.- **4** - Notion of function and derivatives - Theorems on derivatives.- **5** - Derivatives of elementary functions x^n, sin x, cos x, tan x, a^x, log x.- **6**. The exponential function e^x. Natural logarithms - Formula of compound interest - Equivalent and instantaneous rates - Theorem on instantaneous capitalisation.- **7** - Expansions of Taylor and Maclaurin series - Series - Convergence theory - Study of the series corresponding to elementary functions.- **8** - Binomial formula - Binomial series - Convergence.- **9** - Amortisation theory - Amortisation method - Determination of time and rate of interest - Fractional time - Baily formula - Course of amortisation - Table of amortisation.- **10** - Maxima and minima, concavity and convexity of curves - Method of Newton-Fourier for approximate solving of equations - Curvature and curvature-radius.-

V - Teaching mathematical economics

We come now to an incidental activity, which lasted only ten weeks, but availed Broggi to resume his economic studies in Italy and eventually became a landmark in Argentine economic thought. It happened in 1918, the of the University Reform, an event that enhanced trying new approaches. It introduced the so-called 'free teaching', in the way of 'free courses', i. e. courses off the curriculum. In that year Broggi and Gondra introduced Argentine students to 'Pure' or 'mathematical' economics, conducting the very first experience of renewal of economics teaching, through the new way of 'free teaching'.

On 7 June 1918, Broggi and Gondra, professors of statistics and economic history, respectively, reported to the Council of the Faculty of Economic Science his decision to teach a free course in Pure Economics, in ten weekly lectures", in order to "contribute, in the extent of our capabilities, to the diffusion of economic knowledge". The program included: 1.- Object of pure economics. Assumptions of value theory. Economics as an abstract science. 2.- The hedonic principle. Theory of utility. Gossen's theorems on hedonic maxima. Theory of decreasing utility . 3.- Different trends in pure Economics. Stanley Jevons. Walras. The Austrian school. 4. Notion of economic equilibrium. Works of Walras, Edgeworth, Fisher and

Pareto. 5.- Influence of cost on price. Marshall's theorems. Theory of monopoly: Cournot's equation.

The inaugural lecture by Gondra was published under the head of "Pure economics"[13]. In the following year (1919), the Dean himself, Dr Eleodoro Lobos, invited Gondra to teach another quartely- free course. Gondra suggested as program: "Theory of utility. Utility and cost. Gossen's theorems on satisfaction maxima. Demand and supply. Consumer's Rent. Coefficients of production. Interest and rate of discount. Value. General notion of economic equilibrium. International exchanges. Theory of comparative costs. Protectionism and free exchange. Equilibrium and monetary dynamics among open markets. The reading list included Pareto's *Cours*, Walras' *Elements of Pure Economics* , Osorio's *Théorie mathématique de l'échange*, Pantaleoni's *Pure Economics* (in Spanish), and Barone's *Principi*. [14]

Both teachers were complementary, since they came from different disciplines: Broggi from mathematics and economics, Gondra from law and economic history. Although Broggi's handicap was a poor fluency in Spanish, he could read Pareto's *Manuale* in his mother-language, and what is more important, he could deeply understand its Mathematical Appendix.

VI - Existence of the utility function

Ugo Broggi elaborated, as by-product of the course taught with Gondra, a proof of the existence of the utility function, in 'Hedonic Maxima and Indifference curves', published in 1919, when there were no comparable achievements of the like, and by a path analogous to the one contemporarily followed[15] :

> We assume to be reciprocally given the indifference curves M, N that cover continually the area of the plane corresponding to
> $a \geq 0, b \geq 0$ (1)
> which expresses, independently from any definition of utility, that each of them divides the area (1) of the plane into two areas, being one of them such as that our hedonist would never go freely into it (the finite part of the plane enclosed by the indifference curve and the two finite segments cut by it on the co-ordinate axes), while into the other he would always go as far as he is able to do so.

> (Broggi 1919 § 5, 77)

Thus Broggi (1919) showed the existence of the utility function $u(a, b)$ as a continuous and increasing function of the quantities consumed, built up from the indifference curves.[16] His achievement is indicative of Broggi's concern with existence issues, and of his ability for stating and solving them.

VII - On Osorio's book

The book of the Portuguese Paretian follower Antonio Osorio had been included in the reading assignment of Gondra and Broggi's course, but the latter went on working on it and discovered in section § 158 -devoted to building demand as a function of price, according to Pareto- that Osorio reached a result that was only valid for additive utility functions, of the Gossen-Jevons-Walras type, and not for general functions of Edgeworth type. Osorio's flaw, perhaps attributable to his lack of mathematical background (he was a lawyer), was due to not taking into account that the partial derivatives of an *n*-variable function are themselves functions of *n* variables. Broggi's contribution (1921), rather than highlighting Osorio's flaw, was to sketch a sort of Logic of Preference, in which initial, optimal or desired, and excess or

market demands, were sharply distinguished, some twenty years or more before such distinctions became current in economic analysis.

VIII - Existence of general equilibrium

The death of Pareto and Marshall disclosed the differential influence of their works on Argentine academic milieu. While a mere letter of condolence was sent to Cambridge, to honour the memory of the sage of Céligny an important tribute was organised by the UBA. Broggi was entrusted by the Dean J. L. Suárez to speak in the name of professors, and Raúl Prebisch (aged 22) received the same distinction to speak in the name of students. Broggi accepted on 1 September 1923, so that he had scarcely one month to prepare a speech which was delivered on 3 October. Broggi and Prebisch's lectures were published in the October 1923 issue of **Revista de Ciencias Económicas**, the Faculty's economic journal. Broggi availed himself of the homage paid to the last member of the Lausanne school to insert a critique to its founder, more precisely, to Walras' well-known approach to proving the **existence of general equilibrium** by merely counting as much independent equations as unknowns. The short paragraph in which Broggi developed his critique in nonmathematical terms, is the following:

> From the equality of these numbers, the Lausanne school deduces the consequence that the economic problem is determined. This consequence ... seems not to be adequately proven. The algebraic theory of equations systems of arbitrary form does not exist: there only exists a theory of linear equations systems. In the case of linear equations it cannot be asserted that having as much equations as unknowns the latter become determinate. They are if the equations are independent, i. e. if one or several equations of the system cannot be deduced operating on the others, and the theory offers the possibility of investigating in each case if the system's equations are independent or not. It is upon this theory that is founded the examination of whether n relationships or equations with $n + p$ variables $y_1 ... y_n$, $x_1 ... x_p$ define implicitly the n variables y in function of the p variables x, an examination in which, as is known, is specially relevant to consider the functional determinant of the equation-system.

> (Broggi 1923, 149)

In the above text Broggi stated his *critique*. But did he advance any proposal? The following is the short passage that may be taken as a *conjecture* of Wald's approach:

> This consequence [that the economic problem is determined] should be founded on the previous examination of the nature of the economic equations ... Referring to the equations of economics, the necessity of this examination has not even been suspected: would it be done, the examination would perhaps be left without answer, as long as somehow be defined the analytical character of the equations with which to work.

> (Broggi 1923, 149-50)

A few months later (January 1924), a whole issue of the **Giornale degli Economisti e Rivista di Statistica** was devoted to Pareto's life and work, with contributions by Maffeo Pantaleoni, Enrico Barone, Umberto Ricci, Luigi Amoroso, Alfonso de Pietri-Tonelli, Gustavo del Vecchio, Marco Fanno, Ulisse Gobbi, Gino Borgatta, R. Benini, Giuseppe Prato, Roberto Michels, Ettore Circcotti, Giorgio Mortara, and Felipe Vinci. It seems natural to think that Broggi was touched by the publication, and wishful to make known to a wider public what he felt to be an important contribution to general equilibrium analysis, he sent to the Italian journal an abstract of his Buenos Aires address, where the relevant excerpts are the following:

Per la scuola di Losanna il problema della economia è quello di determinare a un tempo le quantità di beni scambiate trasformate risparmiate prestate e il prezzo di ogni bene e del suo uso, e si risolve contando quante sono in complesso le incognite e stabilendo altrettante equazioni.

Della uguaglianza dei due numeri, di equazioni e di incognite, si deduce che il problema economico è determinato e, quindi, che una scienza della economia è possibili.

È questa una conseguenza che dovrebbe essere fondata sull'esame previo della natura delle equazioni della economia, e che non pare ben dimostrata. Esiste una teoria algebrica dei sistemi di equazioni lineari. Nel caso di equazioni lineari no può affermarsi che se esistono tante equazioni quante sono le incognite, queste sono determinate. Lo sono se le equazioni sono indipendenti e la teoria offre la possibilità di investigare se le equazioni di un sistema determinato sono indipendenti, o no. E' su questa teoria che si fonda l'esame di quando n equazioni o relazioni vincolanti le $n + p$ variabili $y_1, y_2, ... y_n, x_1, x_2, ... x_p$ definiscano implicitamente le n y in funzione delle p x, esame rispetto al quale, como è noto, ha tanta importanza la considerazione del determinante funzionale del sistema di relazioni.

Pero quel che concerne l'economia, la necessità di questo esame non è stata neppure sospettata: nè le indagine potrebbe condurre a un risultato qualsiasi finchè non sia definito il carattere delle relazioni, colle quali si opera.

Broggi, in one way or another, performed a great feat. He stated for the first time the modern criticism of Walrasian systems, a non-negligible step in a story that was not written by a single author, but needed the contributions of several scholars. However, as in the previous Broggi's research on other existence issue -that of the utility function- where no proof was intended up to the forties, with regard to the existence of general equilibrium nobody seemed to avail of Broggi's criticism. Pause followed during the twenties. But did Broggi pause? He had given evidence of pursuing an issue for years.[17] He even met the profiles of the people who finally solved the problem: he was a mathematician as Wald, and had tried pure and applied economics as Schlesinger. Why think that Broggi's concern for making known in Europe his criticism to Walrasian systems, was limited to the Italian sphere and his old colleagues at the *Giornale degli Economisti*, and discard the German sphere, where he had so many connections? His paper in *RCE* and *GE*, rather than a mourn for Pareto's death, seems to be indicative of his cognisance of having pointed out some crucial issue. Thus it seems natural to think that he kept concerned for this issue. Given his professional command on modern higher mathematics and economics, it is not stretching imagination to ask whether he produced some attempt of solution, or even tried an exchange of views with German-speaking mathematicians of his acquaintance.

In this connection Baumol and Goldfeld wrote that "Writing in 1929, Robert Remak, a German mathematician, who studied at Göttingen ... seems to have been the first to point out one of the pertinent difficulties ... He showed very clearly that the sign of the solution could not be taken for granted. This immediately suggests a serious gap in Walras' argument".[18]

Activity was resumed over the great slump -an escape from the worries of the time?- by Neisser, Stackelberg, Zeuthen and Schlesinger, who furthered a dissection of Walras' model through the simplified version known as Walras-Cassel model.

Statements by Neisser (1932), Zeuthen (1932) and von Stackelberg (1933)[19] were but fragmentary advances, brought together by Wald (1933-4). Neisser built a Walras-Cassel model that could produce negative solutions for some quantities and prices:

Daraus ergibt sich:

$$p_1 = 5$$
$$p_2 = -20$$
$$q_1 = -35/2$$
$$q_2 = 15/4$$

(Neisser 1932, 424)

Stackelberg "pointed out that if there were fewer commodities than resources, the equations (3) would constitute a set of linear equations with more equations than unknowns and therefore prossess, in general, no solution".[20]

Zeuthen suggested that since a priori it was impossible which resources would remain partially unemployed, the possibility of zero price was to be allotted to those productive resources which remained incompletely used, or at least to the part of them left unused:

Da man nicht im voraus weiß, welche produktiven Leistungen freie Güter sind, sollte man in diesen Gleichungen als letztes Glied einen etwaigen unbenützten Rest einschalten und gleichzeitig als Bedingung festsetzen, daß entweder dieser Rest oder der Preis der Produktions-leistung gleich 0 ist.

(Zeuthen 1933, 2-3)

Schlesinger, from Stackelberg and Neisser results, but unaware of Zeuthen's paper, suggested to introduce dummy variables:

let u_j designate the unutilized excess quantity of input R_j... these m new unknowns $u_1, ..., u_m$ are required to be non-negative... This gives us a modified system of $m + 2n$ equations and m side conditions in $2m + 2n$ unknowns.

(Schlesinger 1933-4, 11)

Wald's paper 'On the Unique Non-Negative Solvability of the New Production Equations' worked out a Walras-Cassel-Schlesinger model, adopting Schlesinger's suggestion. The paper (1933-4) is well-known since its edition by Baumol-Goldfeld (1968). But let us recall the four initial conditions imposed by Wald's proof, as translated into ordinary language:

i. The supplies of the productive resources are strictly positive.
ii. All technological coefficients of production are positive or zero.
iii. In the production of each commodity enters at least one non-negative coefficient of production.
iv. The demand function, $f_j(s_j)$, is defined for every positive value of commodity s_j , and is always non-negative, continuous, and strictly monotone decreasing.

(cf Stigler 1941, 243)

It may be noticed that Wald not only adopted Schlesinger's dummy variables, but also imposed restrictions on technology and upon the analytical character of the demand functions. Their approach was consistent with Broggi's suggestions, namely, starting by 'the previous examination of the nature of the economic equations' and concluding by imposing restrictions on the equations (qualifying 'the analytical character of the equations').

The posterior work by Arrow, Debreu[21] and McKenzie expanded Wald's results, but it is a contemporary story that will not be retold here.

We end this paper with a feeling of success and failure. The former, because some people may have through these lines their first acquaintance with an eminent academician. The latter, because we perceive our attempt to be severely incomplete: we could not find any photo of Broggi, nor the name of his birth-place, his Italian and German studies, his activities

during 1930-1965, his publications in many journals, and so on. Therefore, we invite our fellow researchers, especially those in Italy, to publish if they have any new data on Broggi's life and work, or to get in touch with this author.

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X - Works by Ugo Broggi

ABBREVIATIONS:
GE: *Giornale degli Economisti*
GERS: *Giornale degli Economisti e Rivista di Statistica*
RCE: *Revista de Ciencias Económicas* (Buenos Aires)

1904 Recensione su L.G.Profumo - Le assicurazioni operaie nella legislazione sociale, *GE*, January, 89-90.
1904 Recensione su J.H.Levy - Short Studies in Economic subjects, *GE*, January, 90-1.
1904 Di alcuni problemi intorno alla disoccupazione, *GE*, February, 172-6.
1904 Recensione su Edmond Thery - Situation économique et financière de l'Italie, *GE*, February, 185-6.
1904 Recensione su Eugenio Ambron - Lo scopo e le funzione delle banche di emissione, *GE*, February, 187-8.
1904 Recensione su Alberto De Stefani - Teoria del commercio internazionale, *GE*, March, 282.
1904 Recensione su Eugenio Rignano - Un socialisme en armonie avec la doctrine économique libérale, *GE*, March, 282-3.
1904 Recensione su Domenico Polizio - Il miglior sistema di conversione, *GE*, March, 283.

1904 Di un problema fondamentale di statistica investigatrice, *GE*, May, 392-407.

1904 Recensione su Ugo Tombesi - Malthusianismo e industrialismo, *GE*, May, 470-1.

1904 Recensione su Jacob Strieder - Zur Genesis des modernen Kapitalismus, *GE*, May, 471-2.

1904 Recensione sulla Bibliothèque internationale d'économie politique, *GE*, June, 572-3.

1904 Recensione su Mosely - Des conditions de la vie économique et sociale de lóuvrier aux Etats-Unis, *GE*, July, 95-6.

1904 Recensione de Bertrand Nogaro - La rôle de la monnaie dans le commerce international et la théorie quantitative, *GE*, September, 292-5.

1905 Recensione su Bernard Shaw - The common sense of municipal Trading, *GE*, March, 310-2.

1905 Recensione su Attilio Robiola - Il lavoro ed i suoi effetti sul moderno operaio, *GE*, May, 509-10.

1905 Recensione su Carlo Cassola - I sindacati industriali, *GE*, May, 511-2.

1906 Sui salari agricoli nel Pavese, *GE*, April, 366-72.

1906 *Matematica Attuariale* (Teoria statistica della mortalità. Matematica delle assicurazioni sulla vita) (Milano: Ulrico Hoepli, 1906), 344 p.

1907 Nota su di un problema d'annualità, *GE*, 150-1.

1907 Nota alla statistica metodologica del professore Benini, *GE*, 238-46.

1907 *Die Axiome der Wahrscheinlichkeitsrechnung*. Univ. Göttingen (doctoral dissertation).

1907 *Traité des assurances sur la vie, avec développements sur le calcul des probabilités* (Paris: Herman). Trans. from. It. by S. Lattés, 306 p.

1909 Il teorema della probabilità composta e la definizione descrittiva di probabilitá. *Rendiconti del Circolo Matematico di Palermo*. (Palermo: Tip. Matemática), Vol. 28, 2nd semester, 245-47.

1909 Sur le principe de la moyenne arithmétique. *Enseignement Mathématique*, (Paris: Gauthier-Villars), 11th year, No. 1.

1909 Sur une Intégrale aux Différences. *Enseignement Mathématique*, (Paris: Gauthier-Villars), 11th year, No. 2, March.

1914 *Ecuaciones Integrales Lineales*. En *Contribuciones al Estudio de las Ciencias Físicomatemáticas*. Serie Matemática (La Plata: Universidad Nacional de la Plata).

1915 La curva de los errores. *RCE* , July-August, September.

1916 Sobre los coeficientes de asociación de Yule y de Benini. *RCE*, March-April.

1918 Las reservas de las empresas de seguros sobre la vida en la legislación alemana. *RCE* , May.

1919 Estadística. *RCE* , July.

1919 A Proof of a Theorem of Compound Probabilities. *American Mathematical Monthly*. Vol. 26, Issue 7, September, 188-90.

1919 Máximos hedónicos y líneas de indiferencia. *Anales de la Facultad de Ciencias Económicas*, vol.I, 73-8.

1919 *Análisis Matemático*. (Universidad Nacional de La Plata: Publicaciones de la Facultad de Ciencias Físico-Matemáticas. No. 44. Buenos Aires: Imp. y Casa Edit. Coni), Vol. I, Las nociones fundamentales, December, 152 p. Review of the book in *American Mathematical Monthly*, 1920.

1920 Sobre una determinación de promedio. *RCE*, June-July.

1921 Sobre un teorema de economía matemática. *RCE*, IX, Series II (1), August, 31-3.

1922 Sobre una fórmula de cuadratura, *RCE* July.

1923 Vilfredo Pareto y la teoría del equilibrio económico, *RCE* , XI(27), Series II, October, 141-153.

1924 Sull'economia paretiana, *GE*, June, 331-333.

1924 Sobre un problema de matemática financiera. *RCE*, September-October.

1925 A proposito di «meccanica economica», *GE*, 38-39.

1925 Ancora a proposito di «meccanica economica», *GE*, 498-501.

1925 Ueber mechanische Ausgleichung. *Zeitschrift für angewandte Mathematik und Mechanik*, June.

1926 Sobre interpolación. *RCE*, October.

1927 *Análisis Matemático*. (Universidad Nacional de La Plata: Publicaciones de la Facultad de Ciencias Físico-Matemáticas. No. 81. Buenos Aires: Imp. y Casa Edit. Coni), Vol. II, Teorías generales. Funciones de más de una variable, October, 211 p.

1927 Sobre una nueva fórmula de ajustamiento, *RCE* XV, Series II (72), July, 831-6.

Footnotes

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  fax number: +54 220 4 940283.

2 Cañellas (1963, 192)

3 These publications have still to be checked.

[4] De Cesare (1972, 42), Estévez (1967, 1).

[5] The qualifications appear in de Cesare (1972, 43) and point to a comparison with the work and influence in Argentina of the Spanish Professor of Mathematics Dr Julio Rey Pastor, who did not reach Argentina but after five years of Broggi's appointment at University of La Plata. Rey Pastor's arrival in 1917 is usually deemed as the turning point of mathematical studies in Argentina.

[6] He occupied the seat No. 11, later on occupied by Dr Alejandro M. Unsain and Dr Manuel San Miguel.

[7] Cañellas (1963, 192).

[8] See the list of Founding Members in the back covers of *Revista Matemática*, the Society's journal (1924-7). Cf also Santaló (1961, 380).

[9] The envelope, however, indicates 'Sforza 41, Milan', as Broggi's address.

[10] The letter again indicates the address: 'Milano, Via Franceso Sfoza 41'.

11 The reminiscence is due to Cañellas (1963, 192). His former assistant, E. A. de Cesare (1972, 45) offers a more summary account, erasing the political confession.

[12] This section is based on de Cesare (1972, 34-40), who in turn relied on personal recollections about Broggi, of whom he was assistant from 1923.

6 The article was dedicated: "To my good friend Hugo Broggi".

14 Cf. Pareto, Cours d'économie politique, 2 vols. Lausanne, 1896. L. Walras, Eléments d'économie politique, 4th. ed. Paris-Lausanne, 1900. A. Osorio, Théorie mathématique de léchange, Paris, 1913. M.Pantaleoni, Principios de economía pura, Spanish trans.by L.R.Gondra: Madrid-Buenos Aires, 1918. E. Barone, Principi di economia politica, Rome, 1916.

15 In modern times the properties of order relations, the existence of superior and inferior contour sets, and a weak axiom about the consumption set, are taken as axioms.. Cf., e.g.., A.P. Barten and V. Böhm (1982), p. 384-8.

16 Olivera (1977, 23).

[17] An instance of Broggi's persistence on a theme, is his continuance of interest on compound probabilities (see items for 1909 and 1919 of Broggi's works).

[18] Baumol and Goldfeld (1968, 267)

19 Cf. Arrow and Hahn (1971), Historical introduction.

[20] Arrow and Debreu (1952) [1983].

21 Arrow and Debreu are laureate Nobel Prizes in Economics (1972 and 1983, respectively).

University of Buenos Aires or Universidad de Buenos Aires (UBA) - public higher education institution in Argentina. The history of UBA can be traced back to1821 . The university campus is of urban type and is located in the vicinity of Buenos Aires. University ranking.Â The university is in the International Forum of Public Universities (IFPU). University of Buenos Aires - Official Description. The Universidad de Buenos Aires (UBA) is the biggest university in Argentina, founded on August 12, 1821 in the city of Buenos Aires. It consists of 13 faculties, six hospitals, 10 museums and three high schools: Colegio Nacional de Buenos Aires, Escuela Superior de Comercio Carlos Pellegrini, and Instituto Libre de Segunda EnseÃ±anza. Nuno Fernandes. University of Navarra, IESE Business School; European Corporate Governance Institute (ECGI). Date Written: March 22, 2020. Abstract. This report discusses the economic impact of the Coronavirus/COVID-19 crisis across industries, and countries. It also provides estimates of the potential global economic costs of COVID-19, and the GDP growth of different countries. The current draft includes estimates for 30 countries, under different scenarios. The report shows the economic effects of outbreak are currently being underestimated, due to over-reliance on historical comparisons wit Research outputs, collaborations and relationships for University of Buenos Aires (UBA) published between 1 October 2019 - 30 September 2020 as tracked by the Nature Index.Â Below, the same research outputs are grouped by subject. Click on the subject to drill-down into a list of articles organized by journal, and then by title. Note: Articles may be assigned to more than one subject area. Count.