Theory of Asset Pricing

George Pennacchi

August 2020

Contents

Preface **xiii**

I Single-Period Portfolio Choice and Asset Pricing **1**

1 Expected Utility and Risk Aversion **3**

1.1 Preferences when Returns Are Uncertain 4

1.2 Risk Aversion and Risk Premia 14

1.3 Risk Aversion and Portfolio Choice 25

1.4 Summary 33

1.5 Exercises 34

2 Mean-Variance Analysis **37**

2.1 Assumptions on Preferences and Asset Returns 39

2.2 Investor Indifference Relations 43

2.3 The Efficient Frontier 46

2.3.1 A Simple Example 47

2.3.2 Mathematics of the Efficient Frontier 51

2.3.3 Portfolio Separation 56

2.4 The Efficient Frontier with a Riskless Asset 59

2.4.1 An Example with Negative Exponential Utility 65

2.5 An Application to Cross-Hedging 68
2.6 Summary 72
2.7 Exercises 72

3 CAPM, Arbitrage, and Linear Factor Models 77
3.1 The Capital Asset Pricing Model 79
3.1.1 Characteristics of the Tangency Portfolio 80
3.1.2 Market Equilibrium 82
3.2 Arbitrage 89
3.2.1 Examples of Arbitrage Pricing 91
3.3 Linear Factor Models 95
3.4 Summary 103
3.5 Exercises 104

4 Consumption-Savings and State Pricing 107
4.1 Consumption and Portfolio Choices 109
4.2 An Asset Pricing Interpretation 114
4.2.1 Real versus Nominal Returns 116
4.2.2 Risk Premia and the Marginal Utility of Consumption . . 117
4.2.3 The Relationship to CAPM 118
4.2.4 Bounds on Risk Premia 119
4.3 Market Completeness, Arbitrage, and State Pricing 124
4.3.1 Complete Markets Assumptions 124
4.3.2 Arbitrage and State Prices 126
4.3.3 Risk-Neutral Probabilities 129
4.3.4 State Pricing Extensions 131
4.4 Summary 133
4.5 Exercises 134

II Multiperiod Consumption, Portfolio Choice, and Asset Pricing 139

5 A Multiperiod Discrete-Time Model 141

5.1 **Assumptions and Notation of the Model** 143

5.1.1 Preferences 144

5.1.2 The Dynamics of Wealth 145

5.2 **Solving the Multiperiod Model** 147

5.2.1 The Final Period Solution 148

5.2.2 Deriving the Bellman Equation 151

5.2.3 The General Solution 152

5.3 **Example Using Log Utility** 155

5.4 Summary 161

5.5 Exercises 162

6 Multiperiod Market Equilibrium 167

6.1 Asset Pricing in the Multiperiod Model 168

6.1.1 The Multi-Period Pricing Kernel 169

6.2 The Lucas Model of Asset Pricing 172

6.2.1 Including Dividends in Asset Returns 173

6.2.2 Equating Dividends to Consumption 175

6.2.3 Asset Pricing Examples 176

6.2.4 A Lucas Model with Labor Income 179

6.3 Rational Asset Price Bubbles 182

6.3.1 Examples of Bubble Solutions 184

6.3.2 The Likelihood of Rational Bubbles 185

6.4 Summary 187
6.5 Exercises 188

III Contingent Claims Pricing 191

7 Basics of Derivative Pricing 193
7.1 Forward and Option Contracts 194
7.1.1 Forward Contracts on Assets Paying Dividends 195
7.1.2 Basic Characteristics of Option Prices 198
7.2 Binomial Option Pricing 203
7.2.1 Valuing a One-Period Option 205
7.2.2 Valuing a Multiperiod Option 209
7.3 Binomial Model Applications 213
7.3.1 Calibrating the Model 215
7.3.2 Valuing an American Option 217
7.3.3 Options on Dividend-Paying Assets 223
7.4 Summary 224
7.5 Exercises 225

8 Diffusion Processes and Itô's Lemma 229
8.1 Pure Brownian Motion 231
8.1.1 The Continuous-Time Limit 232
8.2 Diffusion Processes 235
8.2.1 Definition of an Itô Integral 236
8.3 Itô's Lemma 238
8.3.1 Geometric Brownian Motion 241
8.3.2 Kolmogorov Equation 242

8.3.3 Multivariate Diffusions and Itô's Lemma 245
8.4 Summary 247

8.5 Exercises 247

9 Dynamic Hedging and PDE Valuation 251
9.1 Black-Scholes Option Pricing 252
9.1.1 Portfolio Dynamics in Continuous Time 253
9.1.2 Black-Scholes Model Assumptions 257

9.1.3 The Hedge Portfolio 258
9.1.4 No-Arbitrage Implies a PDE 260
9.2 An Equilibrium Term Structure Model 263
9.2.1 A Bond Risk Premium 266
9.2.2 Characteristics of Bond Prices 268

9.3 Option Pricing with Random Interest Rates 270

9.4 Summary 275
9.5 Exercises 276

10 Arbitrage, Martingales, Pricing Kernels 279
10.1 Arbitrage and Martingales 281
10.1.1 A Change in Probability: Girsanov's Theorem 283
10.1.2 Money Market Deflator 286
10.1.3 Feynman-Kac Solution 287
10.2 Arbitrage and Pricing Kernels 288
10.2.1 Linking the Valuation Methods 291

10.2.2 The Multivariate Case 293
10.3 Alternative Price Deflators 294

10.4 Applications 297
10.4.1 Continuous Dividends 297
10.4.2 The Term Structure Revisited 303
10.5 Summary 305
10.6 Exercises 306

11 Mixing Diffusion and Jump Processes 311
11.1 Modeling Jumps in Continuous Time 312
11.2 Itô's Lemma for Jump-Diffusion Processes 314
11.3 Valuing Contingent Claims 316
11.3.1 An Imperfect Hedge 317
11.3.2 Diversifiable Jump Risk 319
11.3.3 Lognormal Jump Proportions 321
11.3.4 Nondiversifiable Jump Risk 323
11.3.5 Black-Scholes versus Jump-Diffusion Model 323
11.4 Summary 326
11.5 Exercises 327

IV Asset Pricing in Continuous Time 329

12 Continuous Time Portfolio Choice 331
12.1 Model Assumptions 333
12.2 Continuous-Time Dynamic Programming 335
12.3 Solving the Continuous-Time Problem 338
12.3.1 Constant Investment Opportunities 340
12.3.2 Changing Investment Opportunities 347
12.4 The Martingale Approach 355

12.4.1 Market Completeness Assumptions 356
12.4.2 The Optimal Consumption Plan 357
12.4.3 The Portfolio Allocation 362
12.4.4 An Example 363

12.5 Summary 368
12.6 Exercises 369

13 Equilibrium Asset Returns **379**

13.1 An Intertemporal Capital Asset Pricing Model 380
13.1.1 Constant Investment Opportunities 381

13.1.2 Stochastic Investment Opportunities 383
13.1.3 An Extension to State-Dependent Utility 386
13.2 Breeden's Consumption CAPM 387
13.3 A Cox, Ingersoll, and Ross Production Economy 391
13.3.1 An Example Using Log Utility 399
13.4 Summary 404
13.5 Exercises 404

14 Time-Inseparable Utility **409**

14.1 Constantinides' Internal Habit Model 411
14.1.1 Assumptions 411
14.1.2 Consumption and Portfolio Choices 416
14.2 Campbell and Cochrane's External Habit Model 421
14.2.1 Assumptions 421
14.2.2 Equilibrium Asset Prices 423
14.3 Recursive Utility 426
14.3.1 A Model by Obstfeld 427

14.3.2 Discussion of the Model 433
14.4 Summary 436
14.5 Exercises 437

V Additional Topics in Asset Pricing 441

15 Behavioral Finance and Asset Pricing 443

15.1 The Effects of Psychological Biases on Asset Prices 446
15.1.1 Assumptions 446
15.1.2 Solving the Model 450
15.1.3 Model Results 454
15.2 The Impact of Irrational Traders on Asset Prices 455
15.2.1 Assumptions 455
15.2.2 Solution Technique 457
15.2.3 Analysis of the Results 462
15.3 Summary 470
15.4 Exercises 471

16 Asset Pricing with Differential Information 473

16.1 Equilibrium with Private Information 474
16.1.1 Grossman Model Assumptions 475
16.1.2 Individuals' Asset Demands 476
16.1.3 A Competitive Equilibrium 477
16.1.4 A Rational Expectations Equilibrium 478
16.1.5 A Noisy Rational Expectations Equilibrium 481
16.2 Asymmetric Information 485

16.2.1 Kyle Model Assumptions 486
16.2.2 Trading and Pricing Strategies 487
16.2.3 Analysis of the Results 491

16.3 Summary 494
16.4 Exercises 494

17 Term Structure Models **499**
17.1 Equilibrium Term Structure Models 500

17.1.1 Affine Models 503
17.1.2 Quadratic Gaussian Models 509

17.1.3 Other Equilibrium Models 512
17.2 Valuation Models for Interest Rate Derivatives 513
17.2.1 Heath-Jarrow-Morton Models 514
17.2.2 Market Models 528

17.2.3 Random Field Models 537
17.3 Summary 543

17.4 Exercises 544

18 Models of Default Risk **547**
18.1 The Structural Approach 548
18.2 The Reduced-Form Approach 553
18.2.1 A Zero-Recovery Bond 554

18.2.2 Specifying Recovery Values 557

18.2.3 Examples 562

18.3 Summary 567

18.4 Exercises 568

Preface

The genesis of this book comes from my experience teaching asset pricing theory to beginning doctoral students in finance and economics. What I found was that no existing text included all of the major theories and techniques of asset valuation that students studying for a Ph.D. in financial economics should know. While there are many excellent books in this area, none seemed ideal as a stand-alone text for a one-semester first course in theoretical asset pricing. My choice of this book's topics were those that I believe are most valuable to someone at the start of a career in financial research. Probably the two features that most distinguish this book from others are its broad coverage and its user-friendliness.

Contents of this book have been used for over a decade in introductory finance theory courses presented to doctoral students and advanced masters students at the University of Illinois at Urbana-Champaign. The book presumes students have a background in mathematical probability and statistics and that they are familiar with constrained maximization (Lagrange multiplier) problems. A prior course in microeconomics at the graduate or advanced undergraduate level would be helpful preparation for a course based on this book. However, I have found that doctoral students from mathematics, engineering, and the physical sciences who had little prior knowledge of economics often are able to understand the course material.

This book covers theories of asset pricing that are the foundation of current theoretical and empirical research in financial economics. It analyzes models of individual consumption and portfolio choice and their implications for equilibrium asset prices. In addition, contingent claims valuation techniques based on the absence of arbitrage are presented. Most of the consumption-portfolio choice models assume individuals have standard, time-separable expected utility functions, but the book also considers more recent models of utility that are not time separable or that incorporate behavioral biases. Further, while much of the analysis makes standard "perfect markets" assumptions, the book also examines the impact of asymmetric information on trading and asset prices. Many of the later chapters build on earlier ones, and important topics reoccur as models of increasing complexity are introduced to address them. Both discrete-time and continuous-time models are presented in a manner that attempts to be intuitive, easy to follow, and that avoids excessive formalism.

As its title makes clear, this book focuses on theory. While it sometimes contains brief remarks on whether a particular theory has been successful in explaining empirical findings, I expect that doctoral students will have additional exposure to an empirical investments seminar. Some of the material in the book may be skipped if time is limited to a one-semester course. For example, parts of Chapter 7's coverage of binomial option pricing may be cut if students have seen this material in a masters-level derivatives course. Any or all of the chapters in Section V also may be omitted. In my teaching, I cover Chapter 15 on behavioral finance and asset pricing, in part because current research on this topic is expanding rapidly. However, if reviewer response is any indication, there are strongly held opinions about behavioral finance and asset pricing, and so I suspect some readers will choose to skip this material all together while others may wish to see it expanded.

Typically, I also cover Chapter 16 which outlines some of the important models of asymmetric information that I believe all doctoral students should know. However, many Ph.D. programs may offer a course entirely devoted to this topic, so that this material could be deleted under that circumstance. Chapters 17 and 18 on modeling default-free and defaultable bond prices contain advanced material that I typically do not have time to cover during a single semester. Still, there is a vast amount of research on default-free term structure models and a growing interest in modeling default risk. Thus, in response to reviewers' suggestions, I have included this material because some may find coverage of these topics helpful for their future research. A final note on the end of chapter problems: most of these problems derive from assignments and exams given to my students at the University of Illinois. The solutions are available for instructor download at the Addison Wesley website.

Acknowledgements

I owe a debt to the individuals who first sparked my interest in financial economics. I was lucky to have been a graduate student at MIT during the early 1980s where I could absorb the insights of great financial economists, including Fischer Black, Stanley Fischer, Robert Merton, Franco Modigliani, Stewart Myers, and Paul Samuelson. Also, I am grateful to my former colleague at Wharton, Alessandro Penati, who first encouraged the writing of this book when we team taught a finance theory course at Università Bocconi during the mid-1990s. He contributed notes on some of the book's beginning chapters.

Many thanks are due to my colleagues and students at the University of Illinois who provided comments and corrections to the manuscript. In addition, I have profited from the valuable suggestions of many individuals from other universities who reviewed drafts of some chapters. I am particularly indebted to the following individuals who provided extensive comments on parts of the book:

Mark Anderson, Gurdip Bakshi, Evangelos Benos, Murillo Campello, David Chapman, Gregory Chaudoin, Michael Cliff, Pierre Collin-Dufresne, Bradford Cornell, Michael Gallmeyer, Christos Giannikos, Antonio Gledson de Carvalho, Olesya Grishchenko, Hui Guo, Jason Karceski, Mark Laplante, Dietmar Leisen, Sergio Lence, Tongshu Ma, Galina Ovtcharova, Kwangwoo Park, Christian Pedersen, Glenn Pedersen, Monika Piazzesi, Allen Poteshman, Peter Ritchken, Saurav Roychoudhury, Nejat Seyhun, Timothy Simin, Chester Spatt, Qinghai Wang, and Hong Yan.

The level of support that I received from the staff at Addison-Wesley greatly exceeded my initial expectations. Writing a book of this scope was a time-consuming process that was made manageable with their valuable assistance. Senior Acquisitions Editor Donna Battista deserves very special thanks for her encouragement and suggestions.

Last but not least my wife Peggy and our triplets George, Laura, and Sally deserve recognition for the love and patience they have shown to me. Their enthusiasm buoyed my spirits and helped bring this project to fruition.

This note introduces asset pricing theory to Ph.D. students in nance. The emphasis is put on dynamic asset pricing models that are built on continuous-time stochastic processes. It is very preliminary. Please let me know if you discover any mistake.Â Preface. i. 1 Introduction to Asset Pricing Theory. 1. 1.1 Basic Abstractions . . . This article compares two leading models of asset pricing: the capital asset pricing model (CAPM) and the arbitrage pricing theory (APT): I argue that while the APT is compatible with the data available for testing theories of asset pricing, the CAPM is not. In reaching this conclusion emphasis is placed on the distinction between the unconditional (relatively incomplete) information which econometricians must use to estimate asset pricing models and the conditional (complete) information which investors use in making the portfolio decisions which determine asset prices. In finance, arbitrage pricing theory (APT) is a general theory of asset pricing that holds that the expected return of a financial asset can be modeled as a linear function of various factors or theoretical market indices, where sensitivity to changes in each factor is represented by a factor-specific beta coefficient. The model-derived rate of return will then be used to price the asset correctlyâ€”the asset price should equal the expected end of period price discounted at the rate implied by the model theory of asset pricing and portfolio management in the discrete time case. Consider the current time t. Let etbe the consumption level (if the. investor bought none of the asset) and denote by Î¾the amount of a certain. kind of asset he chooses to buy with a price of ptper unit. Here the asset. can be bonds, stocks, or options, etc..Â 230 chapter 10. Asset pricing theory. Under this probability measure, we can write the price form as. pt=Et[mt+1(1 + rt)Xt+1.