MORAVA K-THEOIRES OF CLASSIFYING SPACES OF COMPACT LIE GROUPS

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Abstract. This is the abstract for the talk I will give in the 54^{th} Topology Symposium in August 6, 2007 at the University of Aizu. I will talk about the computational aspect of cohomology and generalized cohomology theories of classifying spaces of connected compact Lie groups.

1. Introduction

Let p be a prime number. Let G be a connected compact Lie group and let us denote by BG its classifying space. We say G has p-torsion if and only if $H_*(G;\mathbb{Z})$ has p-torsion. In the case G has no p-torsion, the mod p cohomology of BG is well-known. However, in the case G has p-torsion, the computation of the mod p cohomology is not an easy task. I refer the reader for the book of Mimura and Toda [9] for the detailed account on the cohomology of classifying spaces of compact Lie groups.

I will give the current state of computation of the mod p cohomology theory, Brown-Peterson cohomology and Morava K-theories of classifying spaces of some connected compact Lie groups. The coefficient ring of Brown-Peterson cohomology is

$$BP^* = \mathbb{Z}_{(p)}[v_1, v_2, \ldots, v_n, \ldots],$$

where $\deg v_n = -2(p^n - 1)$ and the coefficient ring of the Morava K-theory $K(n)$ is

$$K(n)^* = \mathbb{Z}/p[v_n, v_n^{-1}].$$

I will describe the results of the joint work with Yagita [2] and some other results obtained after writing of [2]. One of the explicit computational results is as follows:

Theorem 1.1. *For $p = 2$, $G = G_2$, the Morava K-theory $K(n)^*(BG_2)$ of the classifying space of the exceptional Lie group G_2 is given by*

$$K(n)^* \otimes_{BP^*} (BG_2).$$

As an $gr\, BP^*$-module, we have that

$$grBP^*(BG_2)$$

is isomorphic to

$$grBP^*[[y_8, y_{12}, y_{14}]]/(y_{14}\rho_0, y_{14}\rho_{-2}, y_{14}\rho_{-6}) \bigoplus grBP^*[[y_8, y_{12}, y_{14}]]\{w_4\},$$

where the index indicates the degree.

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The computation of the Brown-Peterson cohomology above is done by Kono and Yagita in [6] and the result above is conjectured in [6].

Theorem 1.2. *For $(G,p) = (G_2, 2), (E_6, 2), (F_4, 3), (E_7, 3), (E_8, 5)$ and $(PU(pm), p)$ where $p \not| m$, the E_2-term of the Adams spectral sequence for $P(n)^*(BG)$ has no odd degree elements and it collapses at the E_2-level. In particular, we have*

$$K(n)^*(BG) = K(n)^* \otimes_{BP^*} BP^*(BG)$$

and the Morava K-theory $K(n)^*(BG)$ has no odd degree elements.

Before we begin to deal with the computation, we would like to mention the interpretation of K-theory in terms of representation theory. Recall the following theorem.

Theorem 1.3 (Atiyah-Segal)**.** *There is an isomorphism*

$$R(G)^{\wedge} \to K(BG),$$

where $R(G)^{\wedge}$ is the completion of the complex representation ring of G with respect to the argumention ideal.

Also, Morava $K(0)$-theory is the ordinary cohomology with coefficient in the rational numbers $\mathbb{Q}$ or its p-completion $\mathbb{Q}_p$. Thus, through deRham theory, it is related to differential forms. Morava $K(1)$-theory is related to the p-localization of the complex K-theory and to vector bundles. Morava $K(2)$-theory is related to the elliptic cohomology. Many mathematicians dream of geometric and/or representation theoretical interpretation of Morava K-theories and related cohomology theories, e.g. elliptic cohomology, complex cobordism theory. We hope the computation of Morava K-theories of classifying spaces might shed some light on such geometric and/or representation theoretical interpretation.

2. Ordinary cohomology theory

As we already mentioned, when G has no p-torsion, we have a satisfactory result:

Theorem 2.1. *If G has no p-torsion, then the induced homomorphism*

$$H^*(BG; \mathbb{Z}/p) \to H^*(BT; \mathbb{Z}/p)$$

is a monomorphism. If p is an odd prime, then the image of the above homomorphism is the ring of invariants of the Weyl group W,

$$H^*(BG; \mathbb{Z}/p) = H^*(BT; \mathbb{Z}/p)^W = \mathbb{Z}/p[y_1, \ldots, y_n]$$

where $\deg y_1 \cdots \deg y_n = 2^n |W|$ and $\deg y_1, \ldots, \deg y_n$ are even. In particular, the mod p cohomology of BG is a polynomial algebra over $\mathbb{Z}/p$.

There are compact Lie groups G with p-torsion. The following is the list of p-torsions of simply connected, simple compact Lie groups.

Lie group	2-torsion	3-torsion	5-torsion	p-torsion ($p > 5$)
$SU(n)$	$\times$	$\times$	$\times$	$\times$
$Sp(n)$	$\times$	$\times$	$\times$	$\times$
$\mathrm{Spin}(n)$	$\bigcirc$	$\times$	$\times$	$\times$
G_2	$\bigcirc$	$\times$	$\times$	$\times$
F_4	$\bigcirc$	$\bigcirc$	$\times$	$\times$
E_6	$\bigcirc$	$\bigcirc$	$\times$	$\times$
E_7	$\bigcirc$	$\bigcirc$	$\times$	$\times$
E_8	$\bigcirc$	$\bigcirc$	$\bigcirc$	$\times$

Most of the mod p cohomology theories of classifying spaces of the above Lie groups as graded $\mathbb{Z}/p$-modules are computed. Only the cases $p = 2$, $G = E_8$ and $p = 3$, $G = E_8$ remain unsolved, although some of details are not yet in the literature.

There are important examples of connected compact Lie groups with p-torsion. Among those are the projective classical groups, such as the projective unitary group $PU(n)$ which is the quotient of the unitary group $U(n)$ by its center S^1. The ordinary cohomology theories of projective classical groups seem to be difficult to compute. Among the nontrivial cases, only the cases $p = 2$, $G = PO(4n + 2)$, $PU(4n+2)$, $Sp(4n+2)$ and $p = 3$, $G = PU(3)$ were computed in the 20^{th} century (See [3], [4], [5]). The case $p > 3$, $G = PU(p)$ are computed in [2], [11], recently. The computation of $H^*(PU(p^n); \mathbb{Z}/p)$ $(n \geq 2)$ seems to be difficult.

When G has p-torsion, there are odd degree elements in the mod p cohomology of BG, so that the induced homomorphism

$$H^*(BG; \mathbb{Z}_p) \to H^*(BT; \mathbb{Z}/p)$$

is no longer a monomorphism. Replacing the maximal torus by elementary abelian p-subgroups A's, Quillen proved the following theorem.

Theorem 2.2. *There is an F-isomorphism*

$$H^*(BG; \mathbb{Z}/p) \to \lim_{\leftarrow} H^*(BA; \mathbb{Z}/p).$$

F-isomorphism implies that a power of $x \in \lim\limits_{\leftarrow} H^*(BA; \mathbb{Z}/p)$ is in the image of this homomorphism and each element in the kernel of this homomorphism is nilpotent.

The cohomology of elementary abelian p-subgroups is not only useful in the computation of the mod p cohomology of classifying spaces but also important to our understanding it. So, we hope the following conjecture to be true.

Conjecture 2.3. *For $p > 2$, the induced homomorphism*

$$H^*(BG; \mathbb{Z}/p) \to \prod_A H^*(BA; \mathbb{Z}/p)$$

is a monomorphism where A ranges over the conjugacy classes of elementary abelian p-subgroups of G.

In the case $p = 2$, the above conjecture does not hold. See Kono-Yagita [6].

3. Morava K-theory and Brown-Peterson cohomology theory

The computation of generalized cohomology theories is easy when G has no p-torsion and the coefficient ring of the generalized cohomology theory has no odd degree elements. Since the ordinary cohomology of BG has no odd degree elements, the E_2-term

$$E_2^{p,q} = H^p(BG; E^q)$$

of the Atiyah-Hirzeburch spectral sequence converging to the generalized cohomology theory $E^*(BG)$ collapses at the E_2-level and we have the following proposition.

Proposition 3.1. *The Brown-Peterson cohomology $BP^*(BG)$ is isomorphic to*

$$BP^* \otimes_{\mathbb{Z}_{(p)}} H^*(BG; \mathbb{Z}_{(p)}).$$

The Morava K-theory $K(n)^*(BG)$ is isomorphic to

$$K(n)^* \otimes_{\mathbb{Z}/p} H^*(BG; \mathbb{Z}/p).$$

If G has p-torsion, then the mod p cohomology has an odd degree nonzero element. Our results seem to support the following conjecture.

Conjecture 3.2. *The Brown-Peterson cohomology of the classifying space of a connected compact Lie group has no odd degree elements.*

Many mathematicians believed that this conjecture should be true for all compact Lie groups including all finite groups. But a counterexample was constructed by Kriz in [7], [8]. We do not know any counterexample in the case of connected compact Lie groups.

There is a generalized cohomology theory $P(n)$ for $n \geq 0$. $P(0)$ is BP or its p-completion. We compute the $P(n)$-cohomology in order to compute the Morava K-theories.

Theorem 3.3. *If the induced homomorphism*

$$\rho : BP^*(X) \to P(n)^*(X)$$

is an epimorphism for all $n \geq 0$, then the following hold: for all $n \geq 0$,

$$P(n)^*(BG) \cong P(n)^* \otimes_{BP^*} BP^*(BG)$$

and

$$K(n)^*(BG) \cong K(n)^* \otimes_{BP^*} BP^*(BG).$$

So, if we would like to show that

$$K(n)^*(X) \cong K(n) \otimes_{BP^*} BP^*(X)$$

for all n, it suffices to show that

$$BP^*(X) \to P(n)^*(X)$$

is an epimorphism for all n.

In the case $p = 2$, we have the following results for simply connected simple Lie groups.

$(p = 2)$	G	BP	Morava K
	G_2	○	○
	F_4	○	??
	E_6	○	○
	E_7	??	??
	E_8	??	??

The computation of the Brown-Peterson cohomology above is due to Kono and Yagita in [6]. The computation of Morava K-theories is a new result. Kono and Yagita compute the Brown-Peterson cohomology of $B\mathrm{Spin}(n)$ for $n \leq 10$ and $BPU(4n+2)$, $BSp(4n+2)$, too. Wilson compute the Brown-Peterson cohomology of $BO(n)$ in [12] and Kono and Yagita compute their Morava K-theories in [6]. Recently, Inoue and Yagita compute the Brown-Peterson cohomology and Morava K-theories of $BSO(n)$ for $n \geq 2$ in [1]. In the case $p = 3$, we have the following results for simply connected simple Lie groups.

$(p = 3)$	G	BP	Morava K
	F_4	○	○
	E_6	○	??
	E_7	○	○
	E_8	??	??

The computation of $p = 3$, $G = F_4, PU(3)$ is done in [6]. In the case $p = 3$, $G = E_6$, in [2], we computed the Brown-Peterson cohomology of BE_6, however, we could not conclude

$$K(n)^*(BE_6) = K(n)^* \otimes_{BP^*} BP^*(BE_6).$$

It remains to be an open problem. In the case $p = 5$, we have the following result for simply connected simple Lie groups.

$(p = 5)$	G	BP	Morava K
	E_8	○	○

4. Adams spectral sequence

In [6], they use the Atiyah-Hirzeburch spectral sequence in order to compute the Brown-Peterson cohomology. The Atiyah-Hiruzebruch spectral sequence in [6] does not collapse at the E_2-level. In [2], we use the Adams spectral sequence in order to compute $P(n)$-cohomology and show that the assumption in Theorem 3.3 holds. The Adams spectral sequence is an spectral sequence converging to $P(n)$ cohomology whose E_2-term is given by

$$\mathrm{Ext}_{\mathcal{A}}(H^*(P(n); \mathbb{Z}/p), H^*(BG; \mathbb{Z}/p)) = \mathrm{Ext}_{\mathcal{E}_n}(\mathbb{Z}/p, H^*(BG; \mathbb{Z}/p)),$$

where $\mathcal{A}$ is the mod p Steenrod algebra and $\mathcal{E}_n$ is the subalgebra generated by Milnor operations Q_n, $Q_{n+1}, \dots$. The E_2-term could be computed by taking the (co)homology of the (co)chain complex

$$d_1 : grP(n)^* \widehat{\otimes} H^*(BG; \mathbb{Z}/p) \to grP(n)^* \widehat{\otimes} H^*(BG; \mathbb{Z}/p)$$

where

$$grP(n)^* = \mathbb{Z}/p[v_n, v_{n+1}, \dots,]$$

and

$$d_1(v \otimes x) = \sum_{k=n}^{\infty} vv_k \otimes Q_k x.$$

We compute this (co)chain complex. In this talk, we deal with the case $p = 2$, $G = G_2$, E_6 and give an outline of the computation.

First, we deal with the case $p = 2$, $G = G_2$. Let us recall the mod 2 cohomology of the classifying space of the exceptional Lie group G_2. There is an non-toral elementary abelian 2-subgroup of rank 3, say A, in G_2. The induced homomorphism

$$H^*(BG_2; \mathbb{Z}/2) \to H^*(BA; \mathbb{Z}/2)$$

is a monomorphism and the image of this induced homomorphism is the ring of invariants of general linear group $GL_3(\mathbb{F}_2)$ acting on the polynomial algebra $H^*(BA;\mathbb{Z}/2)$ in the usual manner. So, the mod 2 cohomology of BG_2 is the Dickson invariants

$$D = \mathbb{Z}/2[y_0, y_1, y_2]$$

where $\deg y_k = 2^3 - 2^k$ and through this inclusion, we have the action of the Steenrod algebra, in particular, the action of Milnor operations on $H^*(BG_2;\mathbb{Z}/2)$. Let R be the subalgebra of D generated by y_0^2, y_1^2 and y_2^2. Then D is a free R-module with the basis

$$\{1, y_0, y_0y_1, y_0y_2, y_1, y_2, y_1y_2, y_0y_1y_2\}.$$

Let us write $\beta_0 = y_0^2$, $\beta_1 = y_0y_1$, $\beta_2 = y_0y_2$ and $\beta_3 = y_0$. Then, we have

$$D = R\{1, \beta_1, \beta_2, \beta_3, \frac{\beta_1\beta_2}{\beta_0}, \frac{\beta_1\beta_3}{\beta_0}, \frac{\beta_2\beta_3}{\beta_0}, \frac{\beta_1\beta_2\beta_3}{\beta_0}\}.$$

Proposition 4.1. *There exit $f_1, f_2, f_3 \in grP(n)^*\widehat{\otimes}R$ such that*

$$d_1(\beta_1) = f_1\beta_0, \quad d_1(\beta_2) = f_2\beta_0, \quad d_1(\beta_3) = f_3\beta_0,$$

where $\deg f_1 = 0$, $\deg f_2 = -2$, $\deg f_3 = -6$. *Moreover, f_1, f_2, f_3 is a regular sequence in $grP(n)^*\widehat{\otimes}R$.*

Since R has no odd degree elements and since

$$\deg d_1(\beta_1\beta_2\beta_3/\beta_0^2) = 4,$$

the following proposition completes the proof of Theorem 1.1. The element

$$d_1(\beta_1\beta_2\beta_3/\beta_0^2)$$

corresponds to w_4 in Theorem 1.1.

Proposition 4.2. *Let R be a graded algebra over $\mathbb{Z}/2$ and suppose that R is an integral domain. Let $f_1, \ldots, f_n$ be elements in R. Let z_0 be an element in R and consider R-submodules C_k of*

$$R[z_0^{-1}] \otimes \Delta(\beta_1, \ldots, \beta_n)$$

generated by

$$\frac{\beta_{i_1} \cdots \beta_{i_k}}{z_0^k},$$

where $\Delta(\beta_1, \ldots, \beta_n)$ is a simple system of generators, $1 \leq i_1 < i_2 < \cdots < i_k \leq n$, $k > 0$ and $C_0 = R$. Consider the differential d given by $d(\beta_k) = z_0 f_k$ for $k = 1, \ldots, n$ and by $d(xy) = d(x)y + xd(y)$. If $f_1, \ldots, f_n$ is a regular sequence, then the homology of (C, d) is given by

$$H_0(C, d) = R/(z_0f_1, \ldots, z_0f_n)$$

and

$$H_k(C, d) = \{0\}$$

for $k > 0$.

Next, we deal with the case $p = 2$, $G = E_6$. The mod 2 cohomology $H^*(BE_6;\mathbb{Z}/2)$ is generated by y_4, y_6, y_7, y_{10}, y_{18}, y_{32}, y_{48} and y_{34} with the relations

$$y_7y_{10} = 0, \quad y_7y_{18} = 0, \quad y_7y_{34} = 0, \quad y_{34}^2 = y_{10}^2y_{48} + y_{18}^2y_{32} + \text{lower terms},$$

where the index indicates the degree. There is an elementary abelian 2-subgroup A of rank 5 in $F_4 \subset E_6$. The induced homomorphism

$$H^*(BE_4;\mathbb{Z}/2) \to H^*(BA;\mathbb{Z}/2)$$

is not a monomorphism. The image of this induced homomorphism is isomorphic to

$$H^*(BG_2;\mathbb{Z}/2) \otimes \mathbb{Z}/2[y_{16}^2, y_{24}^2]$$

and the action of Milnor operations on y_{16}^2, y_{24}^2 is trivial. The kernel of this induced homomorphism is the ideal generated by y_{10}, y_{18}, y_{34} and it has no odd degree elements. So, by investigating the long exact sequence of Ext groups induced by the short exact sequence

$$0 \to (y_{10}, y_{18}, y_{34}) \to H^*(BE_6;\mathbb{Z}/2) \to H^*(BG_2;\mathbb{Z}/2) \otimes \mathbb{Z}/2[y_{16}^2, y_{24}^2] \to 0$$

of $\mathcal{E}_n$-modules, we have the collapsing of the Adams spectral sequence.

5. Beyond connected compact Lie groups

There seem to be several directions to extend the homotopy theory of classifying spaces of connected compact Lie groups. One of them is the study of the cohomology of finite Chevalley groups and free loop spaces of classifying spaces. I would like to end this talk with this.

For a connected compact Lie group G, there is a complexification $G(\mathbb{C})$. $G(\mathbb{C})$ is a connected reductive complex algebraic group and there is an reductive integral group scheme $G_{\mathbb{Z}}$ such that $G_{\mathbb{Z}}(\mathbb{C}) = G(\mathbb{C})$ as an algebraic group. Replacing $\mathbb{C}$ by $\mathbb{F}_q$, we have finite Chevalley group $G(\mathbb{F}_q)$. By Friedlander-Quillen theory, if q is a power of p and if ℓ is a prime number not equal to p, then the mod ℓ cohomology of the finite Chevalley group is isomorphic to the mod ℓ cohomology of the following pull-back F_{ϕ^q}:

$$\begin{tikzcd} F_{\phi^q} \arrow[r] \arrow[d] & BG^\wedge \arrow[d, "1\times 1"] \\ BG^\wedge \arrow[r, "1\times\phi^q"] & BG^\wedge \times BG^\wedge, \end{tikzcd}$$

where ϕ^q is the Frobenius map and $BG^\wedge$ is the Bousfield-Kan $\mathbb{Z}/\ell$-completion. If $q-1 \equiv 0$ modulo ℓ, the induced homomorphism

$$1-\phi^{q*} : H^*(BG^\wedge;\mathbb{Z}/\ell) \to H^*(BG^\wedge;\mathbb{Z}/\ell)$$

is zero and the Eilenberg-Moore spectral sequence for $H^*(F_{\phi^q};\mathbb{Z}/\ell)$ has the same E_2-term with the Eilenberg-Moore spectral sequence for $H^*(\mathcal{L}BG;\mathbb{Z}/\ell)$ where $\mathcal{L}BG$ is the free loop space of BG and it is the pull-back of the following diagram.

$$\begin{tikzcd} \mathcal{L}BG \arrow[r] \arrow[d] & BG \arrow[d, "1\times 1"] \\ BG \arrow[r, "1\times 1"] & BG \times BG. \end{tikzcd}$$

If G has no ℓ-torsion, both Eilenberg-Moore spectral sequences collapse at the E_2-level and we have the same mod ℓ cohomology for finite Chevalley groups $BG(\mathbb{F}_q)$ and free loop space $\mathcal{L}BG$. Thus, we have the same mod ℓ cohomology.

On the other hand, the induced homomorphism

$$1 - \phi^{q*} : K(n)^*(BG^\wedge) \to K(n)^*(BG^\wedge)$$

is not zero, wheer $K(n)^* = \mathbb{Z}/\ell[v_n, v_n^{-1}]$. So the E_2-term of the Eilenberg-Moore spectral sequence for $K(n)^*(BG(\mathbb{F}_q))$ differs from the one for $K(n)^*(\mathcal{L}BG)$. Again, when G has no ℓ-torsion, we have a satisfactory answer for Morava K-theories of finite Chevalley groups.

Theorem 5.1 (Tanabe [10])**.** *If G has no ℓ-torsion, then $K(n)^*(BG(\mathbb{F}_q))$ has no odd degree elements.*

When G has an ℓ-torsion, we know little on the mod ℓ cohomology, Brown-Peterson cohomology and Morava K-theories of classifying spaces of finite Chevalley groups $G(\mathbb{F}_q)$ and the free loop space $\mathcal{L}BG$.

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Twisted K-theory of compact Lie groups has been studied by physicists (see e.g. [Br04] [MMS01] [GG04]) as well as by mathematicians, starting with Douglas [Do06]. The computation of the twisted K-groups was extended to Lie groups are not necessarily compact simple and simply con-nected in [GG04][MR17]. The results for twisted K-theory KËšpG, hq, for arbitrary choices of the twist h are already rather complicated and hard to understand.Â The twisted Morava K-homology of all groups in the Whitehead tower of the orthogonal and unitary groups, and their classifying spaces, with the canonical twist, is isomorphic to the underlying untwisted Morava K-homology. This seems to be an instance of a more general phenomenon, and it does occur even in the case of K-homology. In particular, we discuss the classifying spaces BG that are pâ€“compact for all primes when the groups are certain subgroups of simple Lie groups. A survey of the pâ€“compactness of BG for a single prime is included. 55R35; 55P15, 55P60. A pâ€“compact group (see Dwyerâ€“Wilkerson [8]) is a loop space X such that X is Fp â€“nite and that its classifying space BX is Fp â€“complete (see Andersenâ€“Grodalâ€“ MÃ¸llerâ€“Viruel [2] and Dwyerâ€“Wilkerson [11]). We recall that the pâ€“completion of a compact Lie group G is a pâ€“compact group if Ï€0(G) is a pâ€“group. Next, if C(Ï) denotes the centralizer of a group homomorphism MORAVA K-THEORY OF EILENBERG-MAC LANE SPACES ERIC PETERSON This talk is about a 1980s computation by Ravenel and Wilson of the Morava K-theories of certain EilenbergMac Lane spaces.Â The most basic situation is q = 1, so that weâ€™re studying the classifying space H Z/ p j 1 = BZ/ p j . Keep in mind that really the only things we know about Morava K-theory are: (1) Its coefficient ring is Kâˆ— = F p n [vn±], which happens to be a graded field. [p] (2) We also. understand its â€œ p-series,â€ which is the action of the map CPâˆž â†’ CPâˆž on cohomology.Â Facts about these objects are best-organized by a tool called â€œDieudonné theory,â€ which is an analogue of a theory of Lie algebras for p-divisible groups, but I canâ€™t go into that now.