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## **OPTIMAL RESIDENTIAL PROPERTY DEVELOPMENT AND LINEAR PROGRAMMING**

*Ir. Connie Susilawati, Grad. Dipl. Property, M.Com.  
Petra Christian University, Surabaya, Indonesia  
(connie@peter.petra.ac.id)*

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### **ABSTRACT**

*Since the main objective of property developer is to optimise the return from their investment by allocating their scarce resources, linear programming for optimisation is a useful tool to help them to make a sound decision. However, many property developers are not familiar with linear programming approach.*

*Linear programming can be used as a decision making tool to solve allocation problems which is demonstrated herein using a residential development in Surabaya as a case study. Implementing the recommendation which is produced by linear programming will achieve a higher net present value than the actual decisions that have been made using the traditional decision processes.*

*Beside solving developers' problems, linear programming can be extended to accommodate the conflict of interests among developers, the government and consumers. It not only produces a comprehensive decision but also satisfies all participant's objectives.*

## **Introduction**

Applications of linear programming to obtain the solution of land use and property investment problems are scattered in literature. The most common real estate problems require the allocation of scarce resources. For example, allocating different types of uses on one site (Aguilar 1973, Dilmore 1981 and Wu 1989). Applications of linear programming in real estate span a wide range of property types: industrial parks, shopping centres, hotels, apartments and residential development.

Linear programming only produces an optimum solution based on the data that is supplied and the assumptions made. It is highly dependent on the accuracy of the data. Since this study uses the same data which was used for the feasibility study for the same project, the information is assumed to be accurate.

A case study is used to demonstrate data requirements and calculation process to solve the space allocation problems in a residential property development. However, some design constraints such as floor area ratio, building coverage ratio and environmental constraints are omitted since they are assumed to be satisfied by the developer's proposal. The results are compared directly to the actual decisions that have been made. All analyses are carried out on a before tax basis. Although linear programming can carry out sensitivity analysis, the sensitivity results will not be reported in this paper.

## **Linear programming and property investment decisions**

Wu (1985, p. 106) stated that "Mathematical models, when appropriately used, can help decision makers in reaching better decisions which utilise scarce resources more efficiently" than by using subjective judgement. However, modelling will not completely replace the role of personal judgement. The results from mathematical programming models are theoretical solutions, which may require some judgement to be applied in the real world (Eppen, Gould & Schmidt 1993). The user must be aware that this model excludes qualitative factors such as social, political or ethical, which may be very important ones in a particular case.

Eppen, Gould & Schmidt (1993, p. 7) defined linear programming is a constrained optimisation model. A constrained optimisation model is "a problem in which we wish to maximise (or minimise) some function of the decision variables subject to a set of constraints". This integrated model defined explicitly the contributions of all aspects which required a comprehensive study of the problem.

Wu (1989) and Lee (1976) pointed out some important assumptions in a linear programming model: the linear relationships between variables, independent and infinitely divisible variables, and deterministic. Williams (1993) pointed out that in the linear programming model both the objective function and the constraint functions have linear relationships. The alternative course of action must be interrelated through a set of constraints. When the decision variables have to be non-fractional values, integer programming is used (Salkin & Mathur 1989; Lee 1976).

McGeorge (1989, p. 1) mentioned that linear programming is able to solve problems which “can be characterised as having an objective function, variables and constraints”. Linear programming provides optimal solutions to certain kinds of problems by an iteration technique that compares a large number of possible solutions, which can be solved by computer software such as MILP88, MAGIC and MS-Excel (Mouchly & Peiser 1993).

The structure of linear programming model for resource allocation problem, which will be used for this study, is set out below (Lange 1971, pp. 18-22).

**Objective function:** Max: 
$$Z = \sum_{i=1}^m \sum_{j=1}^n w_{ij} X_{ij}$$

**Constraint functions:** Subject to: 
$$\sum_{i=1}^m X_{ij} \leq a_j, (j=1, 2, \dots, n)$$

$$\sum_{j=1}^n X_{ij} \geq b_i, (i=1,2, \dots, m)$$

$$X_{ij} \geq 0 \quad \text{(equations 1)}$$

- Where: Z = objective value  
 w<sub>ij</sub> = profit or cost contribution for allocating resource from i<sup>th</sup> uses to j<sup>th</sup> floor space  
 X<sub>ij</sub> = number of floor space area allocated for i<sup>th</sup> type of uses to j<sup>th</sup> floor  
 a<sub>j</sub> = the amount available for j<sup>th</sup> floor  
 b<sub>i</sub> = the demand required for i<sup>th</sup> type of uses  
 m = number of uses  
 n = number of floor space

Linear programming is a simple model and is suitable for solving real estate problems. Applications of linear programming in real estate are scattered in the design, property management, land use and finance management literature. Linear programming has been utilised in solving real estate problems for both short term and long term investments (Jaffe & Sirmans 1982; Gau & Kohlhepp 1980). A short term investment such as development of a residential project is handled by a developer. On the other hand, an investor wants to receive cash flows and capital appreciation in long term investment such as a shopping centre, an office building, industrial and hotel uses.

Most real estate problems require the allocation of scarce resources such as land, capital, labour force and time, to different type of uses so that developers can achieve the maximum profit. Some estimations of future income and expenditures associated with different type of uses have to be conducted to build the objective function. At the same time, the combinations of different types of uses are expected to satisfy the financial, physical, technological, market and legal constraints (see Table 1).

TABLE 1 SOME COMMON CONSTRAINTS IN THE ALLOCATION PROBLEM

<b>Physical constraints</b>	land area total building area	design criteria utility
<b>Capital constraints</b>	maximum equity maximum loan required rate of return	budget labour force average rent
<b>Legal constraints</b>	town planning regulation such as zoning, floor area ratio and building coverage	parking provision minimum space requirements
<b>Environmental</b>	average daily trip	water usage
<b>Market constraints</b>	absorption rates market demand and preference	proportion of space to other uses minimum space requirement

Sources: Aguilar (1973); Dilmore (1981); McGeorge (1989); Wu (1985 and 1989); Gau and Kohlhepp (1980); Mouchly and Peiser (1993)

Mouchly and Peiser (1993, p. 85) showed that land use optimisation using linear programming may be used to satisfy the objectives of real estate participants. Graaskamp (1981) showed the money flows from three real estate participants, that is, space production group (developers and investors), space consumer group and public infrastructure group. The income which is achieved by each participant needs to cover the cost and profit at a required rate of return. Therefore, when each participant meets his or her objectives, the system achieves an equilibrium at the site concerned: competing claims are resolved (Whipple 1995, p. 46).

Other participants' objectives and preferences will form constraints in optimising the objective function. The objectives and constraints generated by the interactions between the participants in the real estate process are shown in Figure 1. Some constraints are derived from the limitation of each participant's capacity. For example, the consumer objective is to minimise costs such as rents. The acceptable rental rate becomes a constraint for an investor which is determined by market forces. On the other hand, developers will not invest their money if the investment leads to insolvency.

### A residential development in Surabaya

Linear programming will be used to solve space allocation problem in residential development in this section. That the selected project have almost completed confers the advantage of comparing model results with the decision actually taken by the principal concerned. As a consequence, certain aspects such as design and environmental constraints are disregarded.

Company "A" is developing a housing project "X" which was start in 1989. The site area for housing development project "X" which is 362,202.8 square metres or 36.22 hectares. The area is an ideal place for housing development as it is near to amenities like schools, universities, bus port and supermarket.

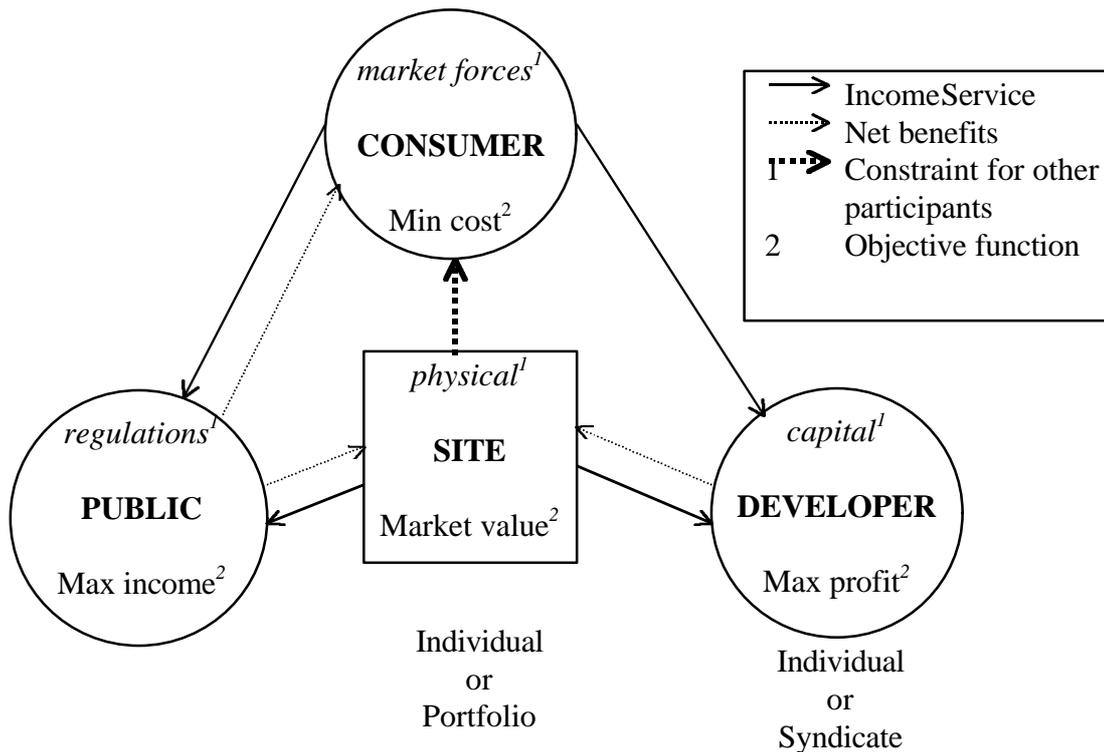


FIGURE 1. THE OBJECTIVES AND CONSTRAINTS GENERATED BY REAL ESTATE MARKET PARTICIPANTS

The developer has to decide the best combination of the type of single houses, shop houses and public facilities which will produce the maximum present value of 22.5 per cent per annum as the required rate of return. This is used as a proxy for the developer’s cost of capital. In order to simplify this case study, eight types of single houses, two types of town houses and two types of shop houses were selected (see Table 2).

The integer variables ( $I_1$  to  $I_{12}$ ) represent the total number of houses. Integer variables are more appropriate than continuous variables, because it is meaningless to have a fractional number of houses. The continuous variables,  $X_{13}$  to  $X_{16}$ , represent areas allocated for public facilities. Therefore, this case uses mixed integer and linear programming models.

Linear programming model

The developer receives income from selling houses and some lots for the international school and hospital. The estimated sales prices in 1992 are shown in Table 2. The developer wants to sell some lots for an international school and a hospital. The land price for both public facilities is about half of the land price for commercial use, which is only Rp 250,000 per square metre (The exchange rate is one US\$ equal to Rp 2,250.00 when this study has been held).

TABLE 2. VARIABLE'S DESCRIPTION

Var.	Type	Description	Building Area (m <sup>2</sup> )	Site Area (m <sup>2</sup> )	Frontage (m)	Depth (m)	Sales price '92 (Rp 000)
I1	single house	T70/136	70	136	8	17	120,700
I2	single house	T70/153	70	153	9	17	128,350
I3	single house	T90/150	90	150	10	15	134,000
I4	single house	T96/160	96	160	8	20	140,600
I5	single house	T144/240	144	240	10	24	193,400
I6	single house	T180/300	180	300	12	25	292,000
I7	single house	T216/360	216	360	12	30	342,400
I8	single house	T270/450	270	450	15	30	418,000
I9	shop house	SH 150/75	150	75	5	15	194,375
			park.space	45	5	9	231,875
I10	shop house	SH 225/75	225	75	5	15	220,000
			park.space	45	5	9	283,250
I11	town house	TH 171/101.75	171	101.75	5.5	18.5	
I12	town house	TH 212/112.75	212	112.75	5.5	20.5	
X13	infrastructure						
X14	landscape						
X15	inter. school						
X16	hospital						

Notes:

I = integer variables which represent number of houses

X = continuous variables which represent total area locate for each public facility

The development process is a gradual one. It started with land acquisition in 1989. The process continued with preparation works such as earthworks, infrastructure, landscape, and utilities. The following stage is the housing development. Some of the more popular housing types are sold within six months. Land for hospital and international school development are sold last. Both construction period and selling phase take 6 to 36 months. However, the selling phase is six months behind the construction period, since it takes six months to complete one house and receives the payment.

Almost 20 per cent of total houses in this project have unfavourable shaped lots. The developer can overcome these problems by allocating the less desirable lots for public facilities and postponing the sales of irregular shaped lots (Kwanda 1996). Therefore, company "A" gives around 15 per cent discount on the irregular lots and they requires one extra year to sell them.

The objective function is to maximise the present value of the net cash flow which is calculated as variable cash inflows less variable cash outflows (see Table 3). House sales prices minus marketing and administration fees are the variable cash inflows. Construction cost, professional fees and contingency costs are considered as variable cash outflows. In order to simplify the calculation, the impact of financial costs and income taxes are ignored. The present value (PV) of net cash flow of each variable is the

coefficient of that variable in the objective function (see equation 2). The result from the present value calculation is the coefficient of objective function for variable  $I_1$ .

Besides the variable cost, the developer needs to pay some fixed costs such as land costs, cut and fill costs, bridge construction costs, utilities installation, and certification costs (see Table 4). The unit costs of land cost and cut and fill are multiplied by total area. The bridge construction cost was about two billion rupiahs (about US\$888,889).

As shown in Table 3, the objective function coefficients are built from the present value of the semi-annual cash flow for each variable. The present value of the net cash flow for each variable and constant is shown as the coefficients of the objective function (see equation 3).

TABLE 3. SEMI-ANNUAL CASH FLOW FOR  $I_1$

month	30	1992 36	42	1993 48	54
<b>Cash Outflows</b>					
Construction cost	10,500	10,500	0	0	0
Professional fees	525	525	0	0	0
Contingencies fees	551	551	0	0	0
Total cost	11,576	11,576	0	0	0
<b>Cash Inflows</b>					
Sales	0	48,280	49,749	11,809	12,160
Marketing fees	0	1,448	1,492	354	365
Administration cost	0	2,342	2,413	573	590
Total net income	0	44,490	45,844	10,882	11,206
<b>Net Cash Flow</b>	-11,576	32,914	45,844	10,882	11,206

$$\text{Objective function} = \sum_{k=1}^m \left( \sum_{j=1}^n \frac{CF_j}{(1+i)^j} \right) X_k \quad (\text{equation 2})$$

Where:  $\sum_{j=1}^n \frac{CF_j}{(1+i)^j}$  = equation for calculating the present value of cash flow

$CF_j$  = net cash flow of  $j^{\text{th}}$  time period

$i$  = required rate of return (in this case is 22.5 per cent per annum)

$j$  = time period, that is one to  $n$

$X_k$  = variable  $k^{\text{th}}$ , that is one to  $m$

Objective function coefficient for variable  $I_1$

$$PV = \frac{-11,576 * I_1}{\left(1 + \frac{0.225}{12}\right)^{30}} + \frac{32,914 * I_1}{\left(1 + \frac{0.225}{12}\right)^{36}} + \frac{45,844 * I_1}{\left(1 + \frac{0.225}{12}\right)^{42}} + \frac{10,882 * I_1}{\left(1 + \frac{0.225}{12}\right)^{48}} + \frac{11,206 * I_1}{\left(1 + \frac{0.225}{12}\right)^{54}} = \mathbf{39,814 I_1}$$

TABLE 4. FIXED COSTS

Description		Unit price
Land acquisition	Rp/m2	12,500
Cut and fill	Rp/m2	6,250
Bridge	Rp	2,000,000,000
Electricity	Rp	1,168,500,000
Water	Rp	1,402,000,000
Phone	Rp	233,650,000
Certification	Rp	1,024,300,000

$$Z = 39,814 I_1 + 45,084 I_2 + 36,600 I_3 + 39,914 I_4 + 60,646 I_5 + 92,840 I_6 + 92,249 I_7 + 116,106 I_8 + 28,277 I_9 + 28,333 I_{10} + 30,469 I_{11} + 40,196 I_{12} - 17 X_{13} - 2 X_{14} + 51 X_{15} + 51 X_{16} - 9,318,753 \quad (\text{equation 3})$$

Three categories of constraints are applied to this model: planning regulation, site area, and marketing preference. The first category is town planning regulations. In Indonesia, developers are required to build a mixture of houses as prescribed in government regulations. Each developer has to provide luxury, medium-cost, and low-cost type houses in the proportion of 1:3:6 (Minister of Home Affairs, Minister of Public Works and Minister of State for People’s Housing 1992). This requirement can be fulfilled in the same or in different locations. On the parcel of land under consideration, the developer decided to develop medium-cost housing only. Therefore, this regulation is not applicable for this project. There is no parking requirement for the housing area, except for flats (Local Government 1992, Appendix 2, p. 1).

Another important planning regulation is that every residential development should have public facilities. In addition, developers can develop only 60 per cent of their land for housing and commercial purposes (Minister of Home Affairs 1987). This requirement restricts the developer to sell less than 60 per cent of the total area. The equation below is used in the linear programming model as the third constraint (see Appendix A). The rest is for public facilities of which the 25 per cent is allocated for infrastructure and 15 per cent is allocated for social facilities such as a school, a hospital and a religious building (Kompas 1996). The distribution of public facilities will be applied as lower and upper bounds (see Appendix A).

$$(Y.3) \quad \sum_{i=1}^{12} I_i * A_i \leq 60\% \text{ site area} \quad (\text{equation 4})$$

The second constraint is the physical restriction. The maximum total site area is 362,202.8 square meters. The equation below is used as the fourth constraint in the linear programming model (see Appendix A).

$$(Y.4) \quad \sum_{i=1}^{12} I_i * A_i + \sum_{j=13}^{17} X_j = 362,202.8 \quad (\text{equation 5})$$

The third category is the market preference constraint. Mrs. R. Yolanda in an interview on 12 December 1995 stated that the proportion between types of houses to total number of houses is usually similar to the government regulation (1:3:6). Therefore, the proportion for bigger houses is 10 per cent, for medium sized houses, 30 per cent and for the smaller houses, 60 per cent of the total houses. Only two constraints from three requirements above are necessary (see equations 6 and 7).

$$(Y.1) \quad \sum_{i=1}^4 I_i * A_i = 0.6 * \sum_{i=1}^8 I_i * A_i \quad (\text{equation 6})$$

$$(Y.2) \quad \sum_{i=7}^8 I_i * A_i = 0.1 * \sum_{i=1}^8 I_i * A_i \quad (\text{equation 7})$$

The last category is the lower and upper bound for each variable. Previous projects are the best guide for making these assumptions. In this case, the site plan was used to assess the range of each variable. The site area allocations in the lower and upper bound need to be converted to number of houses, that is divide total area by area necessary for each variable (see lower and upper bounds in Appendix A). The developer needs to allocate at least five shop houses in this location to serve future residents.

Appendix A shows the complete linear programming model in the format required by MILP88. All fixed costs, which give constant values, are omitted. The input data was given the file name "RBASIC". The optimal solution provided by MILP88 is the maximum present value before considering the constant in the objective function.

### Optimal solution

Table 5 shows the value of the combination of variables. For example, the solution for  $I_1$  shows that 213 houses are provided for the first type of single house (T70/136). While  $X_{13}$  to  $X_{16}$  can be interpreted as area allocated for each use. For instance, the allocation space for infrastructure is 79,685 square metres ( $X_{13}$ ). The net present value of the project is calculated by deducting the constant value from the maximum return result.

Positive net present value shows that the project has more than satisfied developer's required rate of return. However, the objective of this analysis is to find the combination of variables which provide the highest net present value. If there is no change in the given assumptions, the developer will receive a net present value of 53 billion rupiahs. The optimal solution achieved the highest financial return and in the same time satisfied the constraints.

Since the actual site plan has different combinations of housing types, direct comparison is not possible. The problem, however, is simplified by adopting nine categories (see the last column in Table 5). Furthermore, each category will be compared as areas, not number of houses. The integer variables need to be multiplied by area for each type of house in order to be converted to areas.

TABLE 5. THE COMBINATION OF VARIABLES (RBASIC)

Var.	Type		VALUE	New Category
I1	single house	T70/136	213	Small size lots (V <sub>1</sub> )
I2	single house	T70/153	242	
I3	single house	T90/150	72	
I4	single house	T96/160	67	
I5	single house	T144/240	45	Medium size lots (V <sub>2</sub> )
I6	single house	T180/300	252	Large Size lots (V <sub>3</sub> )
I7	single house	T216/360	85	
I8	single house	T270/450	14	
I9	shop house	SH 150/75	5	Shop house (V <sub>4</sub> )
I10	shop house	SH 225/75	5	
I11	town house	TH 171/101.75	30	Town house (V <sub>5</sub> )
I12	town house	TH 212/112.75	20	
X13	infrastructure		79,685	(V <sub>6</sub> )
X14	landscape		36,220	(V <sub>7</sub> )
X15	international school		10,866	(V <sub>8</sub> )
X16	hospital		18,110	(V <sub>9</sub> )

Source: Output from MILP88

**Maximum return** (MILP88 result) = **Rp 62,361,503,630**  
**Fixed costs** (constant in the objective function) = **Rp 9,318,753,000 -**  
**Net present value** = **Rp 53,042,750,630**

Comparison with the actual site plan

The developer did not provide the actual selling period information. Therefore, it is assumed that both RBASIC and actual conditions have the same selling schedule. Furthermore, the same objective function will be used for comparing the actual conditions with the original problem (RBASIC).

One uses the objective function of the original problem with some modifications (see equation 8). Firstly, the average area for each category is calculated. Secondly, the average of objective function’s coefficients for each category is calculated. Thirdly, the new coefficients are calculated by dividing the average objective function’s coefficients by the average area.

$$Z = 265.02 V_1 + 280.19 V_2 + 252.38 V_3 + 228.6 V_4 + 319.26 V_5 - 16.7 V_6 - 2.06 V_7 + 49.81 V_8 + 49.81 V_9 - 9,267,757,990 \quad (\text{equation 8})$$

Table 6 shows combination of total area for each category, objective values and the proportion between commercial purpose and public facilities. Although the actual site plan has a higher percentage for commercial purpose than the linear programming models, the objective value is slightly lower than the RBASIC model.

TABLE 6. COMPARISON OF THE ACTUAL CONDITIONS AND THE LINEAR PROGRAMMING SOLUTIONS

	CATEGORY	SITE PLAN (m <sup>2</sup> )	OPTIMAL SOLUTION (m <sup>2</sup> )
Commercial	Small	48,986	87,514
	Medium	68,766	86,400
	Large	85,401	36,900
	Shophouse	16,127	1,200
	Townhouse	5,953	5,308
Public Facility	Infrastructure	88,116	79,685
	Landscape	31,984	36,220
	Int. school	3,295	10,866
	Hospital	13,575	18,110
	<b>Total</b>	<b>362,203</b>	<b>362,203</b>
	<b>Objective value</b>	<b>Rp 49,425,541,891</b>	<b>Rp 49,453,064,413</b>

<b>Proportion to total area</b>	commercial	62.18%	60.00%
	public facilities	37.82%	40.00%

Young (cited in McGeorge 1989, p. 2) stated that the linear programming optimal solution is a result of what “must be”. For example, theoretically government would not allow a developer to build commercial facilities on more than 60 per cent of land. As consequences the actual allocation might be rejected.

### Conclusion

Linear programming is an integrated model in which the contributions of all aspects are stated explicitly. A comprehensive study by designer, legal, finance and marketing managers contributes to building the objective and constraint functions. A developer will produce a sound development proposal by considering all aspects. Moreover, the risk that developers’ or investors’ proposal will be rejected by government can be reduced (Mouchly & Peiser 1993). Thus, it is more likely that fewer delays caused by amendments to the proposal will be encountered.

Given the results from the above calculation, and the bases of comparison that had to be adopted in the face of information limitations, the conclusion is reached that linear programming can be used as a decision making tool to solve allocation problems. The outcome is that the combination variables' recommendation from linear programming may provide a higher net present value than the traditional approach in which the developer adopted in designing the actual plan. Moreover, the optimal solutions satisfied all the given constraints.

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## APPENDIX A

### LINEAR PROGRAMMING MODEL INPUT

#### RBASIC

	I1	I2	I3	I4	I5	I6	I7	I8	I9	I10	I11	I12	X13	X14	X15	X16		
LOWER	80	71	72	68	45	36	18	14	5	5			72,441	18,110	3,622	10,866		
UPPER									91	91	30	20	108,661	36,220	10,866	18,110		
OBJ	39,814	45,084	36,600	39,914	60,646	92,840	92,249	116,106	28,277	28,333	30,469	40,196	-17	-2	51	51		
Y.1	0.4	0.4	0.4	0.4	-0.6	-0.6	-0.6	-0.6									=	0
Y.2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	0.9	0.9									=	0
Y.3	136	153	150	160	240	300	360	450	120	120	101.75	112.75					<=	217,322
Y.4	136	153	150	160	240	300	360	450	120	120	101.75	112.75	1	1	1	1	=	362,203

This appendix outlines linear programming and its duality relations. Readers are referred to text books such as Gass (1985)<sup>1</sup>, Charnes and Cooper (1961)<sup>2</sup>, Mangasarian (1969)<sup>3</sup> and Tone (1978)<sup>4</sup> for details. More advanced treatments may be found in Dantzig (1963)<sup>5</sup>, Spivey and Thrall (1970)<sup>6</sup> and Nering and Tucker (1993).<sup>7</sup> Most of the discussions in this appendix are based on Tone (1978).

A.1 linear programming and optimal solutions. The simplex method for linear programming starts from a basis, reduces the objective function monotonically by changing bases and finally attains an optimal basis.

Appendix a: linear programming and duality 283.

A.4 dual problem. What is Optimal Design. Value Engineering. Linear Programming. Simplex Analysis using Solver Application Linear Argument: Experimental Architecture. design by developing an optimal solution to design projects, products, and processes. Value engineering is a commonly practiced method of critical design and analysis in construction and manufacturing. It provides both methods and philosophical doctrines for the evaluation and development of building performance to economic solutions. Value model analysis, analyses the performance variables of the designed environment to derive the lowest cost per opportunity optimal solution. This would be defined in common terms, "the best bang for your buck." However, many property developers are not familiar with linear programming approach. Linear programming can be used as a decision making tool to solve allocation problems which is demonstrated herein using a residential development in Surabaya as a case study. Implementing the recommendation which is produced by linear programming will achieve a higher net present value than the actual decisions that have been made using the traditional decision processes. Beside solving developers' problems, linear programming can be extended to accommodate the conflict of interests among developers, the @article{Antunes2018AML, title={A Mixed-integer Linear Programming Model for Optimal Management of Residential Electrical Loads under Dynamic Tariffs}, author={C. Antunes and V. Rasouli and M. J. Alves and Ivaro Gomes}, journal={2018 International Conference on Smart Energy Systems and Technologies (SEST)}, year={2018}, pages={1-6} }. This paper presents a mixed-integer linearprogramming model to automate energy decisions of residential consumers regarding the operation of shiftable, interruptible and thermostatic loads under dynamic tariffs. The cost objective function includes energy costs and a discomfort cost associated with the deviation of indoor temperature with respect to a comfort range.

2) Optimal Substructure: A given problems has Optimal Substructure Property if optimal solution of the given problem can be obtained by using optimal solutions of its subproblems. For example, the Shortest Path problem has following optimal substructure property: If a node  $x$  lies in the shortest path from a source node  $u$  to destination node  $v$  then the shortest path from  $u$  to  $v$  is combination of shortest path from  $u$  to  $x$  and shortest path from  $x$  to  $v$ . The standard All Pair Shortest Path algorithms like Floyd-Warshall. and Bellman-Ford are typical examples of Dynamic Programming. On the other ha